Five Funny Bisimulations

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Standard Bisimulation (given variables P and agents A)

Syntax Language $\mathcal{L}(\Box)$: $\Box_a \varphi$ for 'φ is necessary (for agent a)'.

Structures Model $\mathcal{M} = (S, R, V)$ with pointed model $\mathcal{M}_s$.

Semantics $\mathcal{M}_s \models \Box_a \varphi$ iff $\mathcal{M}_t \models \varphi$ for all t such that $R_a st$.

Bisimulation Relation Z ($\neq \emptyset$) between $\mathcal{M}$ and $\mathcal{M}'$ s.t. for all Zss':

atoms $s \in V(p)$ iff $s' \in V'(p)$;
forth $\forall t$: if $R_a st$, then $\exists t'$ such that $R'_a s' t'$ and Ztt';
back $\forall t'$: if $R'_a s' t'$, then $\exists t$ such that $R_a st$ and Ztt'.

Pointed models are bisimilar iff logically equivalent. (image-fin/sat)

$\mathcal{M}_s \underline{\leftrightarrow} \mathcal{M}'_{s'}$ iff $\mathcal{M}_s \equiv \mathcal{M}'_{s'}$

Example $s : \overline{p} \longrightarrow t : p$ $\qquad u' : p \longleftarrow s' : \overline{p} \longrightarrow t' : p$

Contingency Bisimulation

Syntax Language $\mathcal{L}(\Delta)$

– $\Delta_a\varphi$ for 'φ is non-contingent' (φ is necessarily true or nec. false)
'agent a knows whether φ'

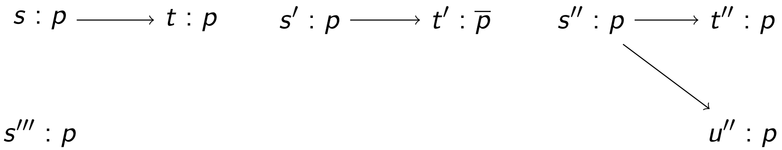
– $\nabla_a\varphi$ for 'φ is contingent' (φ can be both true and false)
'agent a is ignorant about φ'

Semantics

$\mathcal{M}_s \models \Delta_a\varphi$ iff $\forall t, u$ such that $R_a st, R_a su$: $\mathcal{M}_t \models \varphi$ iff $\mathcal{M}_u \models \varphi$

Example

Logically equivalent (but not all standard bisimilar) pointed models



Contingency Bisimulation (single-agent, autobisimulation)

Contingency Bisimulation Relation Z $(\neq \emptyset)$ on $\mathcal{M}$ s.t. for all Zss':

atoms $s \in V(p)$ iff $s' \in V(p)$;

forth if $\exists uv$ such that Rsu, Rsv, and not Zuv, then:
$\forall t$: if Rst, then $\exists t'$ such that $Rs't'$ and Ztt';

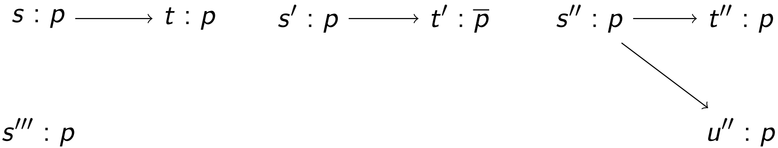
back if $\exists uv$ such that $Rs'u$, $Rs'v$, and not Zuv, then:
$\forall t'$: if $Rs't'$, then $\exists t$ such that Rst and Ztt'.

Results

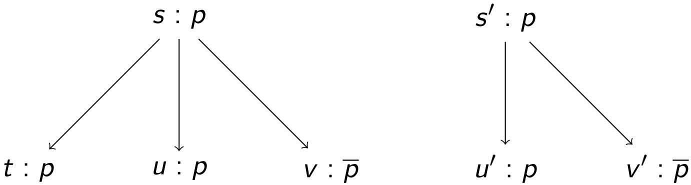
A standard bisimulation is a contingency bisimulation.
Pointed models are contingency bisimilar iff logically equivalent.
(On image-finite / saturated models, in the language $\mathcal{L}(\Delta)$.)
Contingency logic is less expressive than necessity logic.
Almost definability $\nabla\psi \to (\Box\varphi \leftrightarrow \Delta\varphi \wedge \Delta(\psi \to \varphi))$ is valid.

Contingency Bisimulation — Example

Logically equivalent and contingency bisimilar

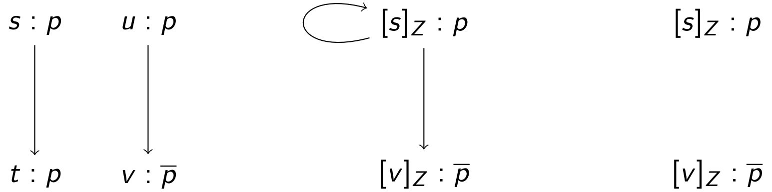


Also logically equivalent and contingency bisimilar



Contingency Bisimulation Contraction

Given a model This is not the contraction But this is



Where $[s]_Z = \{s, t, u\}$ and $[v]_Z = \{v\}$ (and Z is the maximal bisimulation).

Contingency bisimulation contraction $[\mathcal{M}] = ([S], [R], [V])$ def. as

- $[S] = \{[s]_Z \mid s \in S\}$ where $[s]_Z = \{t \in S \mid Zst\}$ (Z is maximal);
- $[R][s][t]$ iff $\exists s't' : Zss', Ztt'$, and $Rs't'$, and $\exists uv : Rs'u, Rs'v$, and not Zuv;
- $[V](p) = \{[s]_Z \mid s \in V(p)\}$.

By taking the reflexivity closure of the relation $[R]$, the bisimulation contraction of an $S5$ model is an $S5$ model.

Contingency Bisimulation — Pubs and People

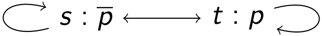
Jie Yanjing

Jie Fan, Yanjing Wang, Hans vD: *Almost Necessary*. Advances in Modal Logic 2014: 178–196.

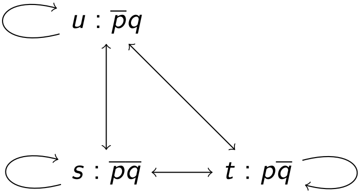
Jie Fan expects to defend his PhD in 2015 at Peking University.

Awareness Bisimulation

Hans is uncertain if there is coffee (p).



Tim informs Hans that coffee and orange juice (q) are not both served.



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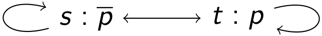

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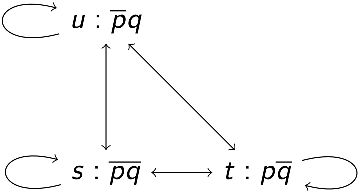
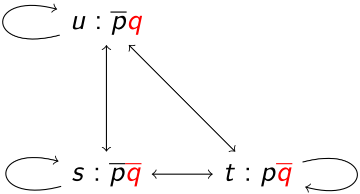




The model before Hans was informed.

Awareness models and explicit knowledge

Syntax $\Box_a\varphi$ for 'agent a implicitly knows φ'
$K^E_a\varphi$ for 'agent a explicitly knows φ'
$A_a\varphi$ for 'agent a is aware of φ'

Structures *Awareness model* $(S, R, \mathcal{A}, V)$ with *awareness function* $\mathcal{A}$ assigning to each state and agent the variables it is aware of.

Semantics $\mathcal{M}_s \models A_a\varphi$ iff $v(\varphi) \subseteq \mathcal{A}_a(s)$
$\mathcal{M}_s \models K^E_a\varphi$ iff $\mathcal{M}_s \models \Box_a\varphi \wedge A_a\varphi$

Example Bisimilar 'for the agent' but not modally equivalent.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

We have that $s \models K^E\Box p$ but $s' \not\models K^E\Box p$.
In states t and t', the agent is unaware of p, thus indifferent to the different value of p in u and u'. We want s and s' to be bisimilar...

Awareness bisimulation

Let $Q \subseteq P$. A Q awareness bisimulation is a collection of binary relations $Z_{Q'}$ between $\mathcal{M}$ and $\mathcal{M}'$ for all $Q' \subseteq Q$ s.t. for all $Z_{Q'}ss'$:

atoms	$\forall p \in Q'$: $s \in V(p)$ iff $s' \in V'(p)$;
aware	$\mathcal{A}_a(s) \cap Q' = \mathcal{A}'_a(s') \cap Q'$;
forth	$\forall t$: if $R_a st$ then $\exists t'$ such that $R'_a s' t'$ and $Z_{Q' \cap \mathcal{A}_a(s)} tt'$;
back	$\forall t'$: if $R'_a s' t'$ then $\exists t$ such that $R_a st$ and $Z_{Q' \cap \mathcal{A}'_a(s')} tt'$.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

Example of a p awareness bisimulation:

$$\begin{array}{rcl} Z_p & = & \{(s,s'),(t,t')\} \\ Z_\emptyset & = & \{(u,u')\} \end{array}$$

Another (maximal) awareness bisimulation, with $Z'_\emptyset \subseteq Z'_p$.

$$\begin{array}{rcl} Z'_p & = & \{(s,s'),(t,t')\} \\ Z'_\emptyset & = & \{(s,s'),(t,t'),(u,u')\} \end{array}$$

Awareness bisimulation and dynamics — Example

Initial models, as before. Awareness bisimilar, and modally equivalent in $\mathcal{L}(K^E)$.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

The agent becomes aware of p.

$$s : p \longrightarrow t : p \longrightarrow u : p \qquad s' : p \longrightarrow t' : p \longrightarrow u' : \overline{p}$$

Clearly the models no longer awareness bisimilar, and $K^E K^E p$ is now a distinguishing formula. Dynamics increases expressivity.

Results — Awareness Logics $\mathcal{L}(A, K^E)$, $\mathcal{L}(A, K^S)$, $\mathcal{L}(A, \Box)$

Speculative knowledge — a novel epistemic operator

$\mathcal{M}_s \models K_a^S \varphi$ iff $\mathcal{M}'_{t'} \models \varphi$ for all t, t' s.t. $R_a st$ and $\mathcal{M}_t \leftrightarroweq_{\mathcal{A}_a(s)} \mathcal{M}'_{t'}$

Explicit $\Rightarrow$ speculative $\Rightarrow$ implicit: $K_a^E \varphi \to K_a^S \varphi$ and $K_a^S \varphi \to \Box_a \varphi$.

K^E: Awareness bisimilarity corresponds to logical equivalence.
K^S: Awareness bisimilarity corresponds to logical equivalence.
$\Box$: Standard bisimilarity corresponds to logical equivalence.

The logics of explicit knowledge and speculative knowledge are equally expressive. The logic of implicit knowledge is more expressive. Adding dynamics makes all 3 logics equally expressive.

Awareness Bisimulation — Pubs and People

Tim

Fer

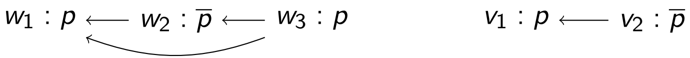
Yi

Hans vD, Tim French, Fernando Velázquez Quesada, Yi N. Wang: *Knowledge, Awareness, and Bisimulation*, Proc. of TARK 2013.

With work unrelated to bisimulation:
Fernando obtained his PhD in 2011 at University of Amsterdam.
Yi obtained his (2nd) PhD in 2013 at University of Bergen.

Plausibility Bisimulation — Example

Plausibility models: equivalence classes encode knowledge, where in each equivalence class the states are ordered into more and less *plausible* states. If s is at least as plausible as t, we write $t \geq s$. (In the picture: an arrow from t to s. We assume reflexive closure.)

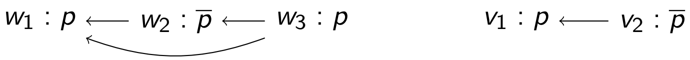


- $K\varphi$: You *know* φ iff φ is true in all possible states.
- $B\varphi$: You *believe* φ iff φ is true in the most plausible states.
- $B^{\psi}\varphi$: You *conditionally believe* φ iff φ is true in the most plausible states satisfying the condition (ψ).
- $\Box\varphi$: You *safely believe* φ iff φ is true cond. to any true restr.

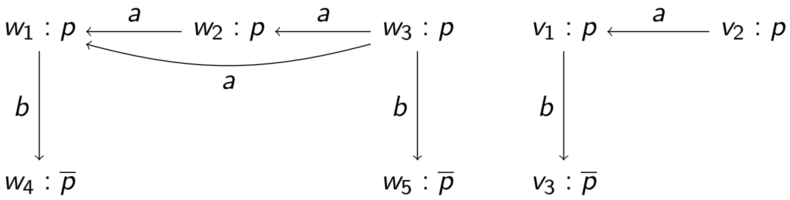
Example $w_1 \models Bp$ but $w_1 \not\models Kp$. $w_1 \models \Box p$ but $w_3 \not\models \Box p$.
The models are logically equivalent in the logics of conditional belief and knowledge. They are not standard bisimilar. A notion of plausibility bisimulation makes them bisimilar. With another semantics for safe belief, they are also logically equiv. in that logic.

Multi-agent example of plausibility bisimilar models

Single-agent: we make models *plausibility bisimilar* by identifying states with the same valuation (with the *most* plausible state).



But in the multi-agent case this no longer works. For example:



In *plausibility bisimulation* the **back** and **forth** clauses refer to the bisimulation in the condition (similar to *contingency bisimulation*).
[forth] clause for Zss': if $s \geq_a^Z t$, $\exists t'$ such that $s' \geq_a^Z t'$ and Ztt';

Results

Results

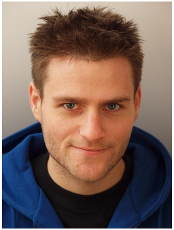
Read the PhD theses of Martin and Mikkel!

Or wait for this to be published in a journal or available on ArXiV.

Plausibility Bisimulation — Pubs and People

Mikkel


Thomas


Martin


Mikkel Birkegaard Andersen, Thomas Bolander, Hans vD, Martin Holm Jensen: *Bisimulation for Single-Agent Plausibility Models*. Australasian AI 2013: 277-288.

Martin obtained his PhD in 2014 at Technical University Denmark.
Mikkel defended his PhD in 2014 at Technical Univ. Denmark.

Refinement

Given this model $\mathcal{M}$

$$\circ \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}$$

It is (standard) bisimilar to the 'blown up' model

$$\bullet \longleftarrow \bullet \longleftarrow \bullet \longleftarrow \circ \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}'$$

A more radical structural transformation is a submodel like

$$\circ \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}''$$

Now consider this: neither a bisimilar copy nor a model restriction.

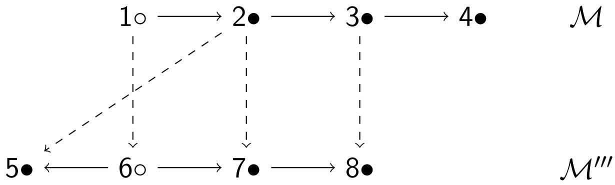
$$\bullet \longleftarrow \circ \longrightarrow \bullet \longrightarrow \bullet \qquad \mathcal{M}'''$$

$\mathcal{M}'''$ is a *refinement* of $\mathcal{M}$: a model restriction of a bisimilar copy.

Refinement — a refinement satisfies back but not forth

A *B refinement* (linking M_s & $M'_{s'}$, notation $M_s \succeq_B M'_{s'}$, where $B \subseteq A$) is a relation $Z_B \subseteq S \times S'$ (containing (s, s')) that satisfies:
— '**atoms**'
— '**back**' for all agents $a \in B$
— '**forth**' and '**back**' for all agents $a \in A \setminus B$

Consider again $\mathcal{M}$ and $\mathcal{M}'''$. Then $\mathcal{M}_1 \succeq \mathcal{M}'''_6$. (Unlabeled.)
The refinement relation is $Z = \{(1,6),(2,5),(2,7),(3,8)\}$.

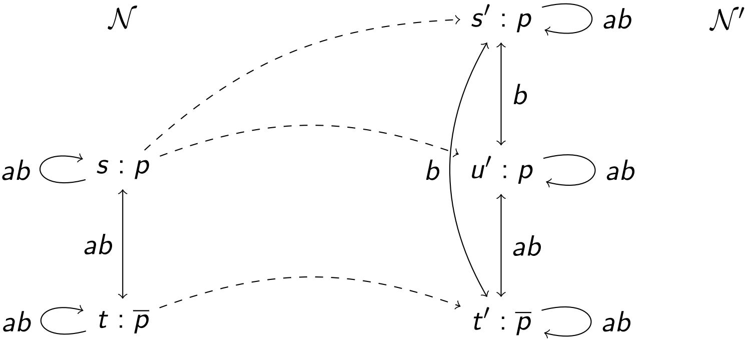


$$\mathcal{M}_s \models \forall_a \varphi \text{ iff for all } \mathcal{M}'_{s'} : \mathcal{M}_s \succeq_a \mathcal{M}'_{s'} \text{ implies } \mathcal{M}'_{s'} \models \varphi$$

$\forall_a \varphi$ is true in a pointed model iff φ is true in all its *a-refinements*.

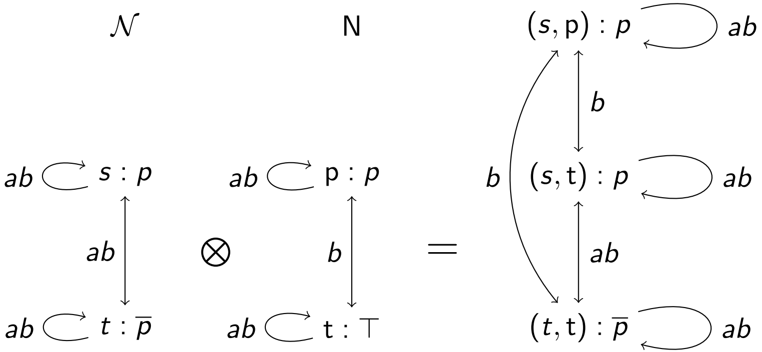
Refinement Modal Logic

Action model execution is a refinement, and vice versa.
Consider $\mathcal{N} \succeq_a \mathcal{N}'$ below.



Refinement Modal Logic

The previous slide depicted $\mathcal{N} \succeq_a \mathcal{N}'$.
Same models, but $\mathcal{N}'$ as $\mathcal{N} \otimes \mathsf{N}$, where N is an action (model).



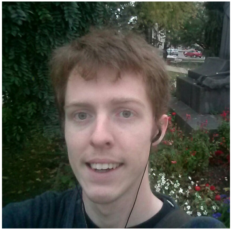
Results for Refinement Modal Logic

- Action model execution is a refinement, and, on finite models, every refinement is the execution of an action model.
- Axiomatization is more elegant if you employ the coalgebraic cover modality instead of the necessity / possibility modalities. $\nabla\{\varphi, \psi\}$ is defined as $\Diamond\varphi \wedge \Diamond\psi \wedge \Box(\varphi \vee \psi)$.
- Refinement modal logic has a complete axiomatization and is equally expressive as multi-agent modal logic.
- Refinement is bisimulation plus model restriction, and refinement quantification is bisimulation quantification followed by relativization: $\exists\varphi$ is equivalent to $\tilde{\exists} q \varphi^q$.
- Refinement epistemic logic (on $S5$ models) has a complete axiomatization.
- Refinement μ calculus is also axiomatized.
  (Future suspects: refinement CTL, refinement PDL, ...)

Refinement — Pubs and People

Laura

James



Sophie

Laura Bozzelli, Hans vD, Tim French, James Hales, Sophie Pinchinat: *Refinement Modal Logic*. Information and Computation 239 (2014) 303–339.

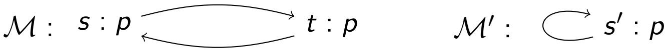
James expects to defend his PhD in 2015 at U o Western Australia.

Bisimulation for Sabotage

Sabotage logic was proposed by Johan van Benthem. A traveller tries to get from A to B by train. The railway operator sabotages (removes links from) the network. It contains an operator for what is true after one removes a pair from the accessibility relation.

$$\mathcal{M}_s \models \langle \mathsf{sb} \rangle \varphi \quad \text{iff} \quad \text{there are } t, u \in S \text{ such that } \mathcal{M}_s^{-tu} \models \varphi$$

where $\mathcal{M}^{-tu}$ is as $\mathcal{M} = (S, R, V)$ except that $R^{-tu} = R \setminus \{(t, u)\}$. The sabotage operation sb is not bisimulation preserving.



We have $\mathcal{M}_s \underline{\leftrightarrow} \mathcal{M}'_{s'}$. But $\mathcal{M}_s \not\models [\mathsf{sb}]\Box\bot$ whereas $\mathcal{M}'_{s'} \models [\mathsf{sb}]\Box\bot$. Correspondence can be regained by strengthening the requirements of bisimulation. Instead of a *standard bisimulation* Z as a relation between states, containing pair (s, s'), a *sabotage bisimulation* is a *relation between state-relation pairs* containing $((s, R), (s', R'))$. Also, we have to add clauses for the dynamic sabotage modality.

Bisimulation for sabotage logic — Pubs and People

Carlos Raul

Carlos Areces, Raul Fervari, Guillaume Hoffmann: *Moving Arrows and Four Model Checking Results*. WoLLIC 2012: 142–153.
Carlos Areces, Hans vD, Raul Fervari, François Schwarzentruber: *Logics with Copy and Remove*. WoLLIC 2014: 51–65.

Raul Fervari obtained his PhD in 2014 at Univ. of Córdoba (Arg.).

H. v Ditmarsch, *Five Funny Bisimulations*. In: S. Ghosh and J. Szymanik (eds), *The Facts Matter. Essays on Logic and Cognition in Honour of Rineke Verbrugge*. College Publications 2015.