Semantic tableau procedure for an iteration-free propositional dynamic logic with parallel composition

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DynRes, septembre 2014, Nancy

PDL [Fischer, Ladner 1977]

- Reason about programs
- $[\alpha]\varphi$ reads "After any application of the program α, φ holds.".

Programs

- A set of atomic programs: $a, b, \dots$
- Sequence of programs: $\alpha ; \beta$
- Test: $\varphi?$
- Iteration: α^*

Language

$$\alpha, \beta := a \mid (\alpha\,;\beta) \mid \varphi? \mid \alpha^*$$
$$\varphi := p \mid \bot \mid \neg\varphi \mid [\alpha]\varphi$$

Semantic

- Kripke model $\mathcal{M} = (W, R, V)$
- $R(\alpha\,;\beta) = R(\alpha) \circ R(\beta)$
- $R(\varphi?) = \{(w, w) \mid \mathcal{M}, w \vDash \varphi\}$
- $R(\alpha^*) = R(\alpha)^*$
- $\mathcal{M}, w \vDash [\alpha]\varphi$ iff $\forall v,\ w\ R(\alpha)\ v \Rightarrow \mathcal{M}, v \vDash \varphi$.

Expressivity

$$\varphi \rightarrow \psi \doteq [\varphi?]\psi$$
$$\langle\alpha\rangle\varphi \doteq \neg[\alpha]\neg\varphi$$

Language

$$\alpha, \beta := a \mid (\alpha \,;\, \beta) \mid \varphi? \mid \alpha^* \mid \alpha^- \mid \alpha \cap \beta$$
$$\varphi := p \mid \bot \mid \neg\varphi \mid [\alpha]\varphi$$

Semantic

- $R(\alpha^-) = \{(y,x) \mid x\, R(\alpha)\, y\}$
- $R(\alpha \cap \beta) = R(\alpha) \cap R(\beta)$

Known facts

- PDL with intersection and converse is *decidable* in the class of all frames. [Lutz 2005]
- PDL with intersection is *undecidable* in the class of frames having at least two deterministic programs. [Harel 1983]
- Iteration-free PDL with intersection and converse is in NEXP-time. [Massacci 2001]

PRSPDL [Benevides, Freitas, Viana 2011]

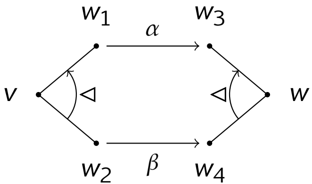
- Reason about *parallel* programs
- Extends PDL with a construct for parallel composition of programs
- The program $\alpha \parallel \beta$ executes α and β simultaneously but on different substates.

Language

$$\alpha, \beta := a \mid (\alpha ; \beta) \mid \varphi? \mid \alpha^* \mid (\alpha \parallel \beta) \mid r_1 \mid r_2 \mid s_1 \mid s_2$$
$$\varphi := p \mid \bot \mid \neg\varphi \mid [\alpha]\varphi$$

Semantic

- Kripke model $\mathcal{M} = (W, R, \lhd, V)$
- $v\ R(\alpha \parallel \beta)\ w$ iff there exists w_1, w_2, w_3 and w_4 such that
$$v \lhd (w_1, w_2)$$
$$w_1\ R(\alpha)\ w_3$$
$$w_2\ R(\beta)\ w_4$$
$$w \lhd (w_3, w_4)$$



Links with BBI

- $\varphi * \psi \doteq \langle \varphi? \parallel \psi? \rangle \top$
- $\varphi \mathrel{-\!\!*_1} \psi \doteq [s_2](\langle r_1 \rangle \varphi \to \psi)$ in $\lhd$-separated frames.
- $\varphi \mathrel{-\!\!*_2} \psi \doteq [s_1](\langle r_2 \rangle \varphi \to \psi)$ in $\lhd$-separated frames.

Definition

A frame $(W, R, \lhd)$ is $\lhd$-*separated* iff

$$x \lhd (y_1, z_1) \wedge x \lhd (y_2, z_2) \Rightarrow y_1 = y_2 \wedge z_1 = z_2$$

Links with PDL with intersection and converse

- $\alpha \parallel \beta \doteq (r_1 ; \alpha ; r_1^-) \cap (r_2 ; \beta ; r_2^-)$ in $\lhd$-separated frames.

Language

$$\alpha, \beta := a \mid (\alpha \,;\, \beta) \mid \varphi? \mid (\alpha \parallel \beta)$$
$$\varphi := p \mid \bot \mid \neg\varphi \mid [\alpha]\varphi$$

Semantic

- Kripke model $\mathcal{M} = (W, R, \lhd, V)$
- $\lhd$-deterministic frames:

$$x_1 \lhd (y, z) \wedge x_2 \lhd (y, z) \Rightarrow x_1 = x_2$$

Subformulae

$$\frac{[\alpha \parallel \beta]\varphi}{\alpha \ldots \quad \beta \ldots \quad \varphi}$$

Subformulae

$$\frac{[\alpha \parallel_i \beta]\varphi}{[\alpha](i,1) \quad [\beta](i,2) \quad \varphi}$$

Language

$$\alpha, \beta := a \mid (\alpha \,;\, \beta) \mid \varphi? \mid (\alpha \parallel_i \beta)$$
$$\varphi := p \mid (i,j) \mid \bot \mid \neg\varphi \mid [\alpha]\varphi$$

Lemma

For all formula φ there is a function F such that

$$[\alpha \parallel_i \beta]\psi \in SF(\varphi) \Rightarrow F(i) = \psi$$

Constraints on formulae

- Each index appears at most once in any formula.
- No placeholders in tests.

Marking function

Given a model $\mathcal{M} = (W, R, \lhd, V)$, a marking function m is a partial function from placeholders to subsets of W.

Semantic

$$\begin{array}{ll} \mathcal{M}, w, m \vDash (i,j) & \text{iff } m(i,j) \text{ is defined and } w \in m(i,j) \\ \mathcal{M}, w, m \vDash [\alpha]\varphi & \text{iff } \forall w',\ w\, R(\alpha)\, w' \Rightarrow \mathcal{M}, w', m \vDash \varphi \\ w\, R(\varphi?)\, w' & \text{iff } w = w' \wedge \mathcal{M}, w, \emptyset \vDash \varphi \end{array}$$

Definition

A formula is *pure* if it contains no placeholder.

Lemma

A pure formula is satisfiable iff the corresponding formula without indices is satisfiable.

Context

Iteration-free PDL with parallel composition

Tableau calculus

- Definition
- Soundness
- Completeness

Decision procedure

Conclusion

Judgement

A judgement about a set W of worlds is either

- $x : \varphi$, where φ is a formula or
- $(x, y) : \alpha$, where α is a program or
- $(x, y, z) : \Delta$, where Δ is either F (forward) or B (backward).

Structure

A structure is a tuple (W, J, K) such that

- W is a set of worlds,
- J is a set of judgements about W,
- $K \subseteq J$ is a subset of *marked* judgements.

Size of programs

$$|a| = 1$$
$$|\varphi?| = 0$$
$$|\alpha\ ;\ \beta| = |\alpha| + |\beta|$$
$$|\alpha \parallel_i \beta| = |\alpha| + |\beta|$$

Lemma

For all $\lhd$-deterministic frame

$$x\ R(\alpha)\ y \text{ and } |\alpha| = 0 \Rightarrow x = y$$

Rules (1/4)

$$\frac{x : [a]\varphi \qquad (x, y) : a}{y : \varphi}$$

$$\frac{x : \langle\alpha\rangle\varphi}{(x, w) : \alpha \qquad w : \varphi}\ |\alpha| \neq 0 \qquad\qquad \frac{x : \langle\alpha\rangle\varphi}{(x, x) : \alpha \qquad x : \varphi}\ |\alpha| = 0$$

$$\frac{x : [\varphi?]\psi}{x : \neg\varphi \;\mid\; x : \psi} \qquad\qquad \frac{(x, x) : \varphi?}{x : \varphi}$$

- w is a fresh world.

Rules (2/4)

$$\frac{x\colon [\alpha\,;\beta]\varphi}{x\colon [\alpha][\beta]\varphi} \qquad \frac{x\colon \langle\alpha\,;\beta\rangle\varphi}{x\colon \langle\alpha\rangle\langle\beta\rangle\varphi}$$

$$\frac{(x,y)\colon \alpha\,;\beta}{(x,w)\colon \alpha \quad (w,y)\colon \beta}\ |\alpha| \neq 0,\ |\beta| \neq 0$$

$$\frac{(x,y)\colon \alpha\,;\beta}{(x,x)\colon \alpha \quad (x,y)\colon \beta}\ |\alpha| = 0 \qquad \frac{(x,y)\colon \alpha\,;\beta}{(x,y)\colon \alpha \quad (y,y)\colon \beta}\ |\beta| = 0$$

- w is a fresh world.

Rules (3/4)

$$\frac{(x,y)\colon \alpha \parallel_i \beta}{(x,w_1,w_2)\colon F \;\; (y,w_3,w_4)\colon B \;\; (w_1,w_3)\colon \alpha \;\; (w_2,w_4)\colon \beta} |\alpha| \neq 0, |\beta| \neq 0$$

$$\frac{(x,y)\colon \alpha \parallel_i \beta}{(x,w_1,w_2)\colon F \;\; (y,w_1,w_3)\colon B \;\; (w_1,w_1)\colon \alpha \;\; (w_2,w_3)\colon \beta} |\alpha| = 0, |\beta| \neq 0$$

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$$\frac{(x,x)\colon \alpha \parallel_i \beta}{(x,w_1,w_2)\colon F \quad (w_1,w_1)\colon \alpha \quad (w_2,w_2)\colon \beta} \; |\alpha| = 0,\ |\beta| = 0$$

- w_1, w_2, w_3 and w_4 are fresh distinct worlds.

Rules (4/4)

$$\frac{x\colon [\alpha \parallel_i \beta]\varphi \qquad (x,y,z)\colon F}{y\colon [\alpha](i,1) \qquad z\colon [\beta](i,2)} \; |\alpha \parallel_i \beta| \neq 0$$

$$\frac{y\colon (i,1) \qquad z\colon (i,2) \qquad (x,y,z)\colon B}{x\colon F(i)}$$

$$\frac{x\colon [\alpha \parallel_i \beta]\varphi}{x\colon \varphi \quad \mid \quad x\colon [\alpha \parallel_i \beta]\bot} \; |\alpha \parallel_i \beta| = 0,\ \varphi \neq \bot$$

$$\frac{x\colon [\alpha \parallel_i \beta]\bot \qquad (x,y,z)\colon \Delta}{y\colon [\alpha]\bot \quad \mid \quad z\colon [\beta]\bot} \; |\alpha \parallel_i \beta| = 0$$

Example

$x_0 : \langle a \rangle \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$

Example

$$x_0 : \langle a \rangle \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$$
$$(x_0, x_1) : a$$
$$x_1 : \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$$

$$\frac{x : \langle \alpha \rangle \varphi}{(x, w) : \alpha \quad w : \varphi} \; |\alpha| \neq 0$$

Example

$x_0 : \langle a \rangle \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$

$(x_0, x_1) : a$

$x_1 : \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$

$(x_1, x_1) : \langle a \parallel_1 b \rangle p?$

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$$\frac{x : \langle \alpha \rangle \varphi}{(x,x) : \alpha \quad x : \varphi} \; |\alpha| = 0$$

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$x_1 : [a \parallel_2 b] q$

$x_1 : \langle a \parallel_1 b \rangle p$

$$\frac{(x, x) : \varphi?}{x : \varphi}$$

Example

$$\begin{aligned}
x_0 &: \langle a\rangle\langle\langle a \parallel_1 b\rangle p?\rangle[a \parallel_2 b]q \\
(x_0, x_1) &: a \\
x_1 &: \langle\langle a \parallel_1 b\rangle p?\rangle[a \parallel_2 b]q \\
(x_1, x_1) &: \langle a \parallel_1 b\rangle p? \\
x_1 &: [a \parallel_2 b]q \\
x_1 &: \langle a \parallel_1 b\rangle p \\
(x_1, x_2) &: a \parallel_1 b \\
x_2 &: p
\end{aligned}$$

$$\frac{x : \langle\alpha\rangle\varphi}{(x, w) : \alpha \quad w : \varphi} \; |\alpha| \neq 0$$

Example

$x_0 : \langle a\rangle\langle\langle a \parallel_1 b\rangle p?\rangle[a \parallel_2 b]q$

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$(x_1, x_1) : \langle a \parallel_1 b\rangle p?$

$x_1 : [a \parallel_2 b]q$

$x_1 : \langle a \parallel_1 b\rangle p$

$(x_1, x_2) : a \parallel_1 b$

$x_2 : p$

$(x_1, x_3, x_4) : F$

$(x_2, x_5, x_6) : B$

$(x_3, x_5) : a$

$(x_4, x_6) : b$

$$\frac{(x, y) : \alpha \parallel_i \beta}{(x, w_1, w_2) : F \;\; (y, w_3, w_4) : B \;\; (w_1, w_3) : \alpha \;\; (w_2, w_4) : \beta} |\alpha| \neq 0, |\beta| \neq 0$$

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$(x_3, x_5) : a$

$(x_4, x_6) : b$

$x_3 : [a](2,1)$

$x_4 : [b](2,2)$

$$\frac{x : [\alpha \parallel_i \beta]\varphi \qquad (x, y, z) : F}{y : [\alpha](i,1) \qquad z : [\beta](i,2)} \; |\alpha \parallel_i \beta| \neq 0$$

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$$(x_1, x_1) : \langle a \parallel_1 b \rangle p?$$
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$$x_1 : \langle a \parallel_1 b \rangle p$$
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$$x_3 : [a](2,1)$$
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$$x_5 : (2,1)$$

$$\frac{x : [a]\varphi \quad (x,y) : a}{y : \varphi}$$

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$x_2 : q$

$$\frac{y : (i,1) \qquad z : (i,2) \qquad (x,y,z) : B}{x : F(i)}$$

Example

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$$x_6 : (2,2)$$
$$x_2 : q$$

Method

A rule is sound iff it preserves interpretability.

Definition (naive)

A function $f : W \longrightarrow W'$ is an interpretation of a structure $\mathcal{S} = (W, J, M)$ in a model $\mathcal{M}' = (W', R', \lhd', V')$ iff

$$x\colon \varphi \in J \Rightarrow \mathcal{M}', f(x), m \vDash \varphi$$
$$(x,y)\colon \alpha \in J \Rightarrow f(x)\ R'(\alpha)\ f(y)$$
$$(x,y,z)\colon \Delta \in J \Rightarrow f(x) \lhd' (f(y), f(z))$$

Difficult rule

$$\frac{y\colon (i,1) \qquad z\colon (i,2) \qquad (x,y,z)\colon B}{x\colon F(i)}$$

Threads

- A thread is defined to be a couple of worlds.
- During the construction of the tableau, a thread is assigned to each world by the thread function t.

How

- $t(x_0) = (x_0, x_0)$
- $t(w) = t(x)$ for the rules

$$\frac{x:\ \langle\alpha\rangle\varphi}{(x,w):\ \alpha \quad w:\ \varphi}\ |\alpha| \neq 0 \qquad \frac{(x,y):\ \alpha\ ;\beta}{(x,w):\ \alpha\ \ (w,y):\ \beta}\ |\alpha| \neq 0,\ |\beta| \neq 0$$

- $t(w_1) = t(w_2) = t(w_3) = t(w_4) = (w_1, w_2)$ for the rules

$$\frac{(x,y):\ \alpha \parallel_i \beta}{(x,w_1,w_2):\ F \quad (y,w_3,w_4):\ B \quad (w_1,w_3):\ \alpha \quad (w_2,w_4):\ \beta}$$

Example

$$x_0 : \langle a \rangle \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$$
$$(x_0, x_1) : a$$
$$x_1 : \langle \langle a \parallel_1 b \rangle p? \rangle [a \parallel_2 b] q$$
$$(x_1, x_1) : \langle a \parallel_1 b \rangle p?$$
$$x_1 : [a \parallel_2 b] q$$
$$x_1 : \langle a \parallel_1 b \rangle p$$
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$$(x_1, x_3, x_4) : F$$
$$(x_2, x_5, x_6) : B$$
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$$(x_4, x_6) : b$$
$$x_3 : [a](2,1)$$
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$$x_6 : (2,2)$$
$$x_2 : q$$

$$t(x_0) = t(x_1) = t(x_2) = (x_0, x_0)$$
$$t(x_3) = t(x_4) = t(x_5) = t(x_6) = (x_3, x_4)$$

Definition

Given

- a structure $\mathcal{S} = (W, J, M)$ with
- a thread function $t : W \longrightarrow W^2$,
- a model $\mathcal{M}' = (W', R', \lhd', V')$,
- a function $f : W \longrightarrow W'$ and
- a function $g : W^2 \longrightarrow \mathcal{P}(W')^{\mathbb{N}\times\{1,2\}}$,

(f, g) is an interpretation of $\mathcal{S}$ in $\mathcal{M}'$ iff

$$x : \varphi \in J \Rightarrow \mathcal{M}', f(x), g(t(x)) \vDash \varphi$$
$$(x, y) : \alpha \in J \Rightarrow f(x)\ R'(\alpha)\ f(y)$$
$$(x, y, z) : \Delta \in J \Rightarrow f(x) \lhd' (f(y), f(z))$$

$$x' \lhd' (y', z') \wedge y' \in g(x, y)(i, 1) \wedge z' \in g(x, y)(i, 2) \Rightarrow \mathcal{M}', x', g(x, y) \vDash F(i)$$

Definition

A tableau is fair iff whenever a rule is applicable at some node on any branch, all the conclusions of this rule belong to another node on the same branch.

Proposition

If a fair tableau for φ is open then φ is satisfiable.

Proof

From a structure $\mathcal{S} = (W, J, K)$ we define the model $\mathcal{M} = (W, R, \triangleleft, V)$ and the marking function m as

$$\begin{aligned} R(a) &= \left\{(x,y) \in W^2 \,\middle|\, (x,y)\colon a \in J\right\} \\ \triangleleft &= \left\{(x,y,z) \in W^3 \,\middle|\, \exists \Delta \in \{F,B\},\ (x,y,z)\colon \Delta \in J\right\} \\ V(p) &= \{x \in W \mid x\colon p \in J\} \\ m(i,j) &= \{x \in W \mid x\colon (i,j) \in J\} \end{aligned}$$

Extension rules

$$\frac{x \colon \langle\alpha\rangle\varphi}{(x,w) \colon \alpha \qquad w \colon \varphi}\ |\alpha| \neq 0$$

$$\frac{(x,x) \colon \alpha \parallel_i \beta}{(x,w_1,w_2) \colon F \qquad (w_1,w_1) \colon \alpha \qquad (w_2,w_2) \colon \beta}\ |\alpha| = 0,\ |\beta| = 0$$

Definition

A rule instance π is *appropriate* for a structure $\mathcal{S} = (W, J, K)$ and a world $x \in W$ iff π is applicable to $\mathcal{S}$ and either π is not an extension rule or π's premises involve only x.

Procedure TABLEAUX

Input: A structure $\mathcal{S} = (W, J, M)$ and a world $x \in W$.
Result: A final structure $\mathcal{S}_f = (W_f, J_f, M_f)$.
Data: A set J_0 of judgements and a structure $\mathcal{S}' = (W', J', M')$.

$J_0 \leftarrow \{\mathfrak{j} \in J \mid \mathfrak{j}$ involves only $x\}$
$\mathcal{S}' \leftarrow (W, J_0, M \cap J_0)$
while *there is a rule's instantiation* π *appropriate to* $\mathcal{S}'$ *and* x
do
    $\mathcal{S}' \leftarrow$ a successor of $\mathcal{S}'$ by π
$\mathcal{S}_f \leftarrow (W', J \cup J', M \cup M')$
foreach $y \in W' \setminus W$ **do**
    $\mathcal{S}_f \leftarrow$ TABLEAUX$(\mathcal{S}_f, y)$

Lemma

After the saturation loop, no judgement involving only one current world can be further obtained by applying the inference rules.

Proposition

The TABLEAUX *procedure is fair.*

Proposition

The Tableaux *procedure gives a PSPACE decision procedure for iteration-free PDL with parallel composition interpreted on* $\lhd$*-deterministic frames.*

Proof

- At each iteration of the procedure, the saturation loop creates a polynomial number of new worlds.
- The recursive depth of the procedure is bounded by a polynomial.
- Each iteration of the procedure needs only polynomial memory space.

Proposition

The validity problem for the iteration-free PDL with parallel composition interpreted on $\triangleleft$-deterministic frames is PSPACE-complete.

Future works

- Add the iteration.
- Add the store and restore programs.
- Adapt the method to $\triangleleft$-separated frames.