# A dynamic epistemic logic with separation
# - some remarks -

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ANR DynRes Meeting - Nancy

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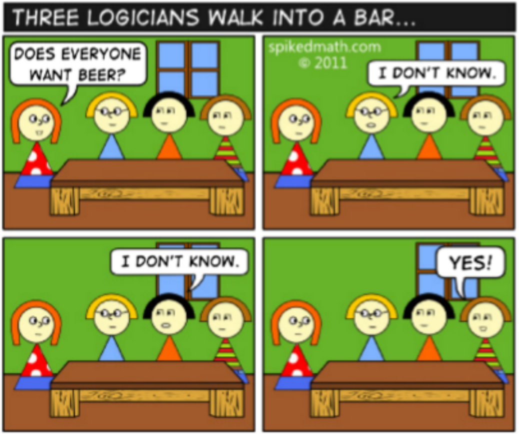
# Introduction - Epistemic logic




- How the third logician knows that all logicians want a beer?
- How to model the logician knowledge evolution?

# Epistemic logic - Semantics

**Epistemic logic - language / model**

- A set of agents ($A$) and a set of propositional symbols (Prop)
- Language:
$$X ::= p \mid \neg X \mid X \wedge X \mid K_a X$$
such that $p \in \text{Prop}$ and $a \in A$
- Model: $\mathcal{M} = (S, \{\sim_a\}_{a \in A}, V)$
  - $S$ is a non empty set of *states*
  - $\sim_a \subseteq S \times S$ are equivalence relations
  - $V : \text{Prop} \to \mathbb{P}(S)$ are equivalence relations

# Epistemic logic - Semantics

**Epistemic logic - semantics**

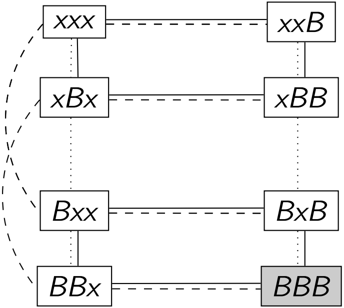
- $s \models_{\mathcal{M}} p$ iff $s \in V(p)$
- $s \models_{\mathcal{M}} \neg\phi$ iff $s \not\models_{\mathcal{M}} \phi$
- $s \models_{\mathcal{M}} \phi \wedge \psi$ iff $s \models_{\mathcal{M}} \phi$ and $s \models_{\mathcal{M}} \psi$
- $s \models_{\mathcal{M}} K_a\phi$ iff $\forall s' \in R \cdot s \sim_a s' \Rightarrow s' \models_{\mathcal{M}} \phi$

- $K_a\phi$: "the agent $a$ knows that $\phi$"
- $\widehat{K}_a\phi \equiv \neg K_a \neg\phi$

# Epistemic logic - An example


- $A = \{A_1, A_2, A_3\}$
- $S = \{xxx, xxB, xBx, xBB, Bxx, BxB, BBx, BBB\}$
- $s_1 \sim_{A_i} s_2$ iff $\lfloor s_1 \rfloor_i = \lfloor s_2 \rfloor_i$, where $\lfloor s \rfloor_i$ is the $i$th character of $s$ ($\lfloor xBx \rfloor_1 = x$ and $\lfloor xBx \rfloor_2 = B$)
- $V(P_{allbeer}) = \{BBB\}$

# Epistemic logic - An example


solid line: $\sim_{A_1}$

dashed line: $\sim_{A_2}$

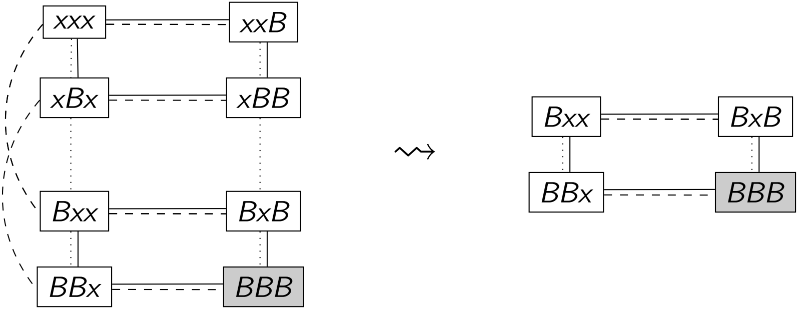
dotted line: $\sim_{A_3}$

The grey node forces $P_{allbeer}$

# Epistemic logic - An example


$$\neg(K_{A_1} P_{allbeer} \vee K_{A_1} \neg P_{allbeer})$$

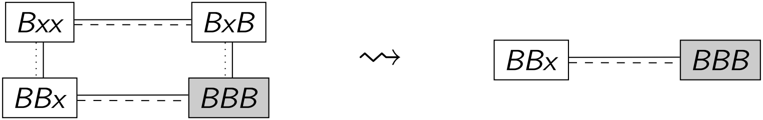
We remove the states that do not satisfy the formula (for instance, $xBx \models K_{A_1} \neg P_{allbeer}$):



# Epistemic logic - An example


$$\neg(K_{A_2} P_{allbeer} \vee K_{A_2} \neg P_{allbeer})$$

We remove the states that do not satisfy the formula (for instance, $Bxx \vDash K_{A_2} \neg P_{allbeer}$):



# Epistemic logic - An example


$K_{A_3} P_{allbeer}$

We remove the states that do not satisfy the formula:

$\boxed{BBx}$ - - - - - - - - $\boxed{BBB}$ $\rightsquigarrow$ $\boxed{BBB}$

- We can remark that the states are beers, that can be viewed as *resources* (composition/decomposition)
- Extension of epistemic logic with the *separation* that allows the states to be really considered as resources

# BI - Resource logic

**Bunched Implications (BI) logic** (O'Hearn-Pym 1999, Pym 2002)

- $\textbf{BI} = \begin{cases} \wedge, \vee, \rightarrow, \top, \bot \text{ (additives)} \\ *, -\!\!*, \mathrm{I} \text{ (multiplicatives)} \end{cases}$

  **BI** (intuitionistic additives) , **BBI** (classical additives)

- Sequents with bunches (trees of formulae where internal nodes are "," or ";"): $\dfrac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi -\!\!* \psi}$ $\dfrac{\Gamma; \phi \vdash \psi}{\Gamma \vdash \phi \rightarrow \psi}$

- Bunches can be viewed as areas of a model:

  $A, (B; C), A \rightsquigarrow$ [ $A$ ¦ $BC$ ¦ $A$ ]

- Resources are areas and propositional symbols are properties of resources (areas)

- **BI** and **BBI** focus on separation (, and $\wedge$) / sharing (; and $*$)

# BI - Logical kernel for resource logics

**Separation logics**

- **BI** and **BBI** logical kernels of separation logics
- Some separation logics:
  - **PL**: Pointer (Separation) Logic with $(x \mapsto a, b)$ (O'Hearn et al. 2001)
  - **BI-Loc**: Separation Logic with locations (Biri-Galmiche 2007)
  - **MBI**: Separation Logic with modalities for processes $(R, E \xrightarrow{a} R', E')$ (Pym-Tofts 2006)
  - **DBI**: Separation Logic with modalities for dynamic properties of resources (Courtault-Galmiche 2013)

▶ **Extension of the epistemic logic with BBI logic**

# Plan

# Plan

# Epistemic logic with separation (ELS) - Language

**Language**

- ELS = BBI + Epistemic logic:

  $\phi ::= p \mid \mathrm{I} \mid \neg X \mid X \wedge X \mid X \vee X \mid X \rightarrow X \mid X * X \mid X \mathrel{-\!\!*} X \mid K_a X$

- An interesting modality:

$$K_a^{\phi}\psi \equiv \phi \mathrel{-\!\!*} (K_a\psi)$$

"A resource that satisfies $\phi$ allows the agent $a$ to know $\psi$"

# Epistemic logic with separation (ELS) - Models

**Model**

- Partial resource monoid (PRM) is a structure $\mathcal{R} = (R, \bullet, e)$:
  - $R$ is a set of *resources*
  - $e \in R$
  - $\bullet : R \times R \rightharpoonup R$ such that, for any $r_1, r_2, r_3 \in R$:
    - Neutral element: $r_1 \bullet e \downarrow$ and $r_1 \bullet e = r_1$
    - Commutativity: if $r_1 \bullet r_2 \downarrow$ then $r_2 \bullet r_1 \downarrow$ and $r_1 \bullet r_2 = r_2 \bullet r_1$
    - Associativity: if $r_1 \bullet (r_2 \bullet r_3) \downarrow$ then $(r_1 \bullet r_2) \bullet r_3 \downarrow$ and $r_1 \bullet (r_2 \bullet r_3) = (r_1 \bullet r_2) \bullet r_3$
- A *model* is a triple $\mathcal{M} = (\mathcal{R}, \{\sim_a\}_{a \in A}, V)$ such that:
  - $\mathcal{R} = (R, \bullet, e)$ is a PRM
  - For all $a \in A$, $\sim_a \subseteq R \times R$ is an equivalence relation
  - $V : \mathrm{Prop} \to \mathbb{P}(R)$ is a *valuation*

# DMBI Logic - Semantics

## Semantics

- Let $\mathcal{M} = (\mathcal{R}, \{\sim_a\}_{a \in A}, V)$ be a model:
  - $r \vDash_{\mathcal{M}} p$ iff $r \in V(p)$
  - $r \vDash_{\mathcal{M}} \top$ always
  - $r \vDash_{\mathcal{M}} \bot$ never
  - $r \vDash_{\mathcal{M}} \mathrm{I}$ iff $r = e$
  - $r \vDash_{\mathcal{M}} \phi * \psi$ iff $\exists r_1, r_2 \in R \cdot r_1 \bullet r_2 \downarrow$ and $r = r_1 \bullet r_2$ and $r_1 \vDash_{\mathcal{M}} \phi$ and $r_2 \vDash_{\mathcal{M}} \psi$
  - $r \vDash_{\mathcal{M}} \phi \mathrel{-\!*} \psi$ iff $\forall r' \in R \cdot (r \bullet r' \downarrow$ and $r' \vDash_{\mathcal{M}} \phi) \Rightarrow r \bullet r' \vDash_{\mathcal{M}} \psi$
  - $r \vDash_{\mathcal{M}} K_a\phi$ iff $\forall r' \in R \cdot r \sim_a r' \Rightarrow r' \vDash_{\mathcal{M}} \phi$
- Validity:

  $\phi$ is *valid* iff $r \vDash_{\mathcal{M}} \phi$ for all resources $r$ of all models $\mathcal{M}$

# Plan

# ELS - The example revisited

Three agents $A_1$, $A_2$ and $A_3$ are in a bar.

In this bar, the customers are only allowed to order at most two beers. The waiter explains to the agents that he can not carry more than four beer glasses.

He asks them if he will be able to bring them their order, in other words he asks if the agents want four beers or less.
- $A_1$ answers that he is not able to answer yes or no.
- Then, $A_2$ answers also that he does not know.
- Finally, $A_3$ answers that the waiter will be able to bring their order.

Thus the waiter asks how many glasses he has to bring.
- $A_1$ answers that he wants only one beer.
- Then $A_2$ says that if $A_3$ would order two more beers then the waiter would not be able to bring the glasses.
- Hearing that, $A_3$ answers that they want three beers: one for $A_1$, two for $A_2$ and none for himself.

# ELS - The example revisited

**The model**

- $A = \{A_1, A_2, A_3\}$
- $\mathcal{R} = (R, \bullet, e)$:
  - $R = \{{B_1}^i {B_2}^j {B_3}^k \mid i, j, k \in \{0, 1, 2\}\}$, where ${B_1}^2 {B_2}^1 {B_3}^0$ means "$A_1$ orders two beers, $A_2$ orders one beer and $A_3$ orders none"
  - The resource composition:

$$ {B_1}^{i_1} {B_2}^{j_1} {B_3}^{k_1} \bullet {B_1}^{i_2} {B_2}^{j_2} {B_3}^{k_2} = \begin{cases} \uparrow & \text{if} \quad i_1 + i_2 > 2 \text{ or } j_1 + j_2 > 2 \\ & \qquad \text{or } k_1 + k_2 > 2 \\ {B_1}^{i_1+i_2} {B_2}^{j_1+j_2} {B_3}^{k_1+k_2} & \text{otherwise} \end{cases} $$

  - Obviously: $e = {B_1}^0 {B_2}^0 {B_3}^0$

# ELS - The example revisited

## The model

- The equivalence relations:

$$B_1{}^{i_1} B_2{}^{j_1} B_3{}^{k_1} \sim_{A_1} B_1{}^{i_2} B_2{}^{j_2} B_3{}^{k_2} \quad \text{iff} \quad i_1 = i_2$$

$$B_1{}^{i_1} B_2{}^{j_1} B_3{}^{k_1} \sim_{A_2} B_1{}^{i_2} B_2{}^{j_2} B_3{}^{k_2} \quad \text{iff} \quad j_1 = j_2$$

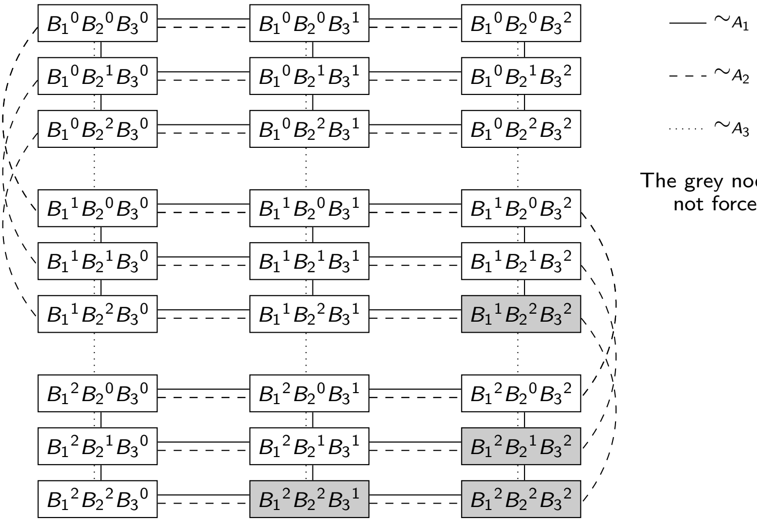
$$B_1{}^{i_1} B_2{}^{j_1} B_3{}^{k_1} \sim_{A_3} B_1{}^{i_2} B_2{}^{j_2} B_3{}^{k_2} \quad \text{iff} \quad k_1 = k_2$$

- Prop $= \{P_1, P_2, P_3, H\}$
  - $V(P_1) = \{B_1{}^1 B_2{}^0 B_3{}^0\}$
  - $V(P_2) = \{B_1{}^0 B_2{}^1 B_3{}^0\}$
  - $V(P_3) = \{B_1{}^0 B_2{}^0 B_3{}^1\}$
  - $V(H) = \{B_1{}^i B_2{}^j B_3{}^k \mid i + j + k \leqslant 4\}$

  - $P_i$: the order contains only on beer for $A_i$
  - $H$: the waiter can hold the order

# ELS - The example revisited

The waiter: "will I be able to hold your order?"

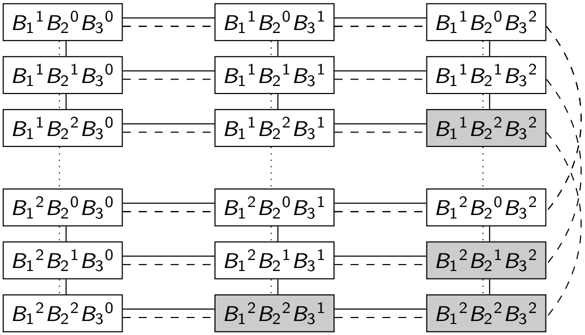



The grey nodes do not force $H$

# ELS - The example revisited

$A_1$: "I don't know."

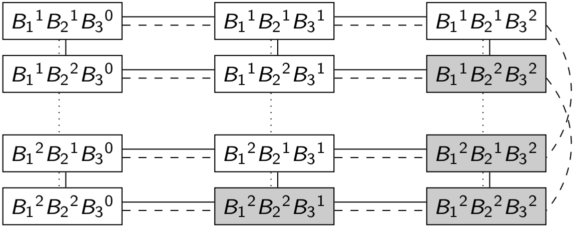
$\rightsquigarrow$ All resources that do not satisfy $\neg K_{A_1} H \wedge \neg K_{A_1} \neg H$ are hidden:



# ELS - The example revisited

$A_2$: "I don't know."

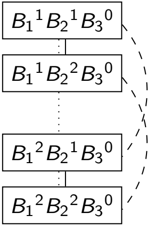
⇝ All resources that do not satisfy $\neg K_{A_2} H \wedge \neg K_{A_2} \neg H$ are hidden:



# ELS - The example revisited

$A_3$: "Yes."

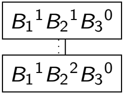
$\rightsquigarrow$ All resources that do not satisfy $K_{A_3}H$ are hidden:



# ELS - The example revisited

$A_1$: "I want one beer"

$\rightsquigarrow$ All resources that do not satisfy $P_1 * \neg(P_1 * \top)$ are hidden:



# ELS - The example revisited

$A_2$: "if $A_3$ would order two more beers then the waiter would not be able to bring the glasses"

$\rightsquigarrow$ All resources that do not satisfy $K_{A_2}((P_3 * P_3) \multimap \neg H)$ are hidden:

$$\boxed{{B_1}^1 {B_2}^2 {B_3}^0}$$

Now, $A_3$ knows that $A_1$ wants one beer, $A_2$ wants two beers and $A_3$ wants no beer:

$$K_{A_3}(P_1 * P_2 * P_2)$$

# Plan

## Conclusion / Works in progress

- Epistemic logic with separation:
  - Semantics / expressiveness
  - Tableaux calculus with countermodel extraction
- Public announcement logic with separation:
  - Semantics / expressiveness
  - Tableaux calculus with countermodel extraction
- Other modal extensions:
  - A la DBI (dynamic properties of resources)
  - A la DMBI (action performing)