Modelling of concurrent processes in DMBI logic

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Introduction - resource logics

Resources

- Resource is a key notion in computer science:
  - Memory
  - Processes
  - Messages
- Different concerns about resources:
  - Location
  - Ownership
  - Access to
  - Consumption of

▶ **Study of resources and related notions through logics**

Introduction - resource logics

Bunched Implications (BI) logic (O'Hearn and Pym 1999, Pym 2002)

- **BI** $= \left\{ \begin{array}{l} \wedge, \vee, \rightarrow, \top, \bot \text{ (additives)} \\ *, -\!\!*, \mathrm{I} \text{ (multiplicatives)} \end{array} \right.$

  BI (intuitionistic additives) , **BBI** (classical additives)

- Sequents with bunches (trees of formulae where internal nodes are "," or ";"): $\dfrac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi -\!\!* \psi}$ $\dfrac{\Gamma; \phi \vdash \psi}{\Gamma \vdash \phi \rightarrow \psi}$

- Bunches can be viewed as areas of a model:

  $A, (B; C), A \quad \rightsquigarrow$

- Resources are areas and propositional symbols are properties of resources (areas)

- **BI** and **BBI** focus on separation (,) / sharing (;)

Introduction - resource logics

Separation logics

- **BI** and **BBI** logical kernels of separation logics
- Some separation logics:
  - **PL**: Pointer (Separation) Logic with $(x \mapsto a, b)$ (O'Hearn et al. 2001)
  - **BI-Loc**: Separation Logic with locations (Biri-Galmiche 2007)
  - **MBI**: Separation Logic with modalities for processes $(R, E \xrightarrow{a} R', E')$ (Pym-Toft 2006)
  - **DBI**: Separation Logic with modalities for dynamic properties of resources (Courtault-Galmiche 2013)

▶ **Study of dynamics in resource/separation logics**

Introduction - resource logics

Dynamics in resource logics

- What are systems with dynamic resources?
  - Systems that transform resources (producers / consumers)
  - Systems that modify resource properties (value of cells of a cellular automata): no resource production/consumption
- Resource logics and dynamics
  - **BI**: Properties on resources = no dynamics
  - **MBI** $(R, E \xrightarrow{a} R', E')$: Dynamics is resource transformation
  - **DBI** (**BI** $+ \Diamond, \Box$): Dynamic properties of resources

Introduction - MBI logic

MBI and **SCRP** (Pym-Tofte 2006)

- **SCRP**: Synchronous Calculus of Resources and Processes
  - Processes: $E ::= 0 \mid X \mid a : E \mid E + E \mid E \times E \mid \nu R.E \mid fix_i X.E$
  - **SCRP** transitions (some rules):

$$\frac{}{R, a : E \xrightarrow{a} \mu(a, R), E}\ (\mu(a,R)\downarrow) \qquad \frac{R, E \xrightarrow{a} R', E' \qquad S, F \xrightarrow{b} S', F'}{R \circ S, E \times F \xrightarrow{a\#b} R' \circ S', E' \times F'}\ (R \circ S \downarrow)$$

- **MBI**: **BI**/**BBI** + modalities ($\langle a \rangle$, $[a]$, $\langle a \rangle_\nu$, $[a]_\nu$)
- Forcing relation:
  - $R, E \vDash \phi * \psi$ iff $\exists R_1, R_2, E_1, E_2 \cdot R = R_1 \circ R_2$ and $E \sim E_1 \times E_2$ and $R_1, E_1 \vDash \phi$ and $R_2, E_2 \vDash \psi$
  - $R, E \vDash \langle a \rangle \phi$ iff $\exists R', E' \cdot R, E \xrightarrow{a} R', E'$ and $R', E' \vDash \phi$
  - $R, E \vDash \langle a \rangle_\nu \phi$ iff $\exists T, R', E' \cdot R \circ T, E \xrightarrow{a} R', E'$ and $R', E' \vDash \phi$

Introduction - MBI logic

An example: mutual exclusion

- Processes:

  $E \overset{def}{=} nc : E + critical : E_{critical}$
  $E_{critical} \overset{def}{=} critical : E_{critical} + critical : E$

- Minimum resources required for the action: $\rho(nc) = \{e\}$ and $\rho(critical) = \{R\}$

- The μ function: $\mu(a, R) = R$ for any a action

- The action $critical\#critical$ is never performed: $R, E \times E \vDash [critical\#critical]\perp$

- Remarks:
  - Only a calculus with bunches and without completeness
  - $R, E \times E \vDash [critical\#critical]\perp$ does not mean that in any reachable state, couple (resource, process), it is impossible to execute two concurrent critical actions (**need of $\Diamond$ and $\Box$**)

Introduction - DBI logic

DBI logic

- Dynamic modal **BI**
  - **BI** with modalities $\Diamond$ and $\Box$
  - Dynamic resource properties
  - A calculus that is sound and complete
- **DBI** models:
  - a resource monoid: resources
  - a graph: states and a state preorder (reachability)
- Forcing relation:
  - $r, s \vDash \phi * \psi$ iff $\exists r_1, r_2 \cdot r_1 \bullet r_2 \sqsubseteq r$ and $r_1, s \vDash \phi$ and $r_2, s \vDash \psi$ (remark: $*$ separates only the resource r)
  - $r, s \vDash \Diamond\phi$ iff $\exists s' \cdot s \preceq s'$ and $r, s' \vDash \phi$

Introduction - DBI logic

An example: properties on states of webservices

- A set of composed webservices $W = \{W_0, W_1, W_2, W_3, \ldots\}$
- A model:

	t_0	t_1	t_2	t_3	t_4	t_5
W_0 :	id	ru	ru	ru	id	id
W_1 :	id	id	ru	ru	ru	id

....

(id: idle)

(ru: running)

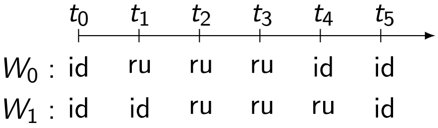
- An interpretation $[\![.]\!]$:
  - $[\![P_{idle}]\!] = \{(S, t_i) \mid \exists W_i \in S \cdot W_i$ is *idle* at time $t_i\}$
  - $[\![P_{running}]\!] = \{(S, t_i) \mid \exists W_i \in S \cdot W_i$ is *running* at time $t_i\}$

where $S \subseteq W$ is a set of webservices.

For example: $S, t \models P_{idle}$ if there is at least a webservice in S that is idle at time t

Introduction - DBI logic

An example: properties on states of webservices



	t_0	t_1	t_2	t_3	t_4	t_5
W_0 :	id	ru	ru	ru	id	id
W_1 :	id	id	ru	ru	ru	id

(id: idle)
(ru: running)

....

- Properties that can be expressed:
  - $\{W_0, W_1\}, t_1 \vDash P_{idle}$
  - $\{W_0, W_1\}, t_1 \vDash P_{idle} \wedge P_{idle}$ but $\{W_0, W_1\}, t_1 \nvDash P_{idle} * P_{idle}$
  - $\{W_0, W_1\}, t_0 \vDash P_{idle} * P_{idle}$
  - $\{W_0, W_1\}, t_0 \vDash (P_{idle} * P_{idle}) \wedge \Diamond(P_{idle} * P_{running})$
- Remark: resource transformation cannot be express in **DBI** (it is not possible to model the messages that are produced / exchanged by the webservices)

Introduction - results

Some results

- **DMBI** logic
  - captures resource transformation ($\approx$ **MBI**)
  - includes modalities $\Diamond$ and $\Box$ ($\approx$ **DBI**)
  - restriction to only one process ($\not\approx$ **MBI**)
- Semantics: μ-dynamic resource monoids
- Expressiveness: **DMBI** models can capture *n* concurrent processes that manipulate resources (but no production of processes $\not\approx$ **MBI**)
- Proof theory: a tableaux method that is sound and complete
- Counter-model extraction

Plan

Plan

DMBI Logic - Language

Language

- **DMBI** $=$ **BBI** $+ \ \langle a \rangle \ [a] \ \Diamond \ \Box$:

  $$\phi ::= p \mid \bot \mid \mathrm{I} \mid \phi \to \phi \mid \phi * \phi \mid \phi \mathrel{-\!\!*} \phi \mid \langle a \rangle \phi \mid [a]\phi \mid \Diamond\phi \mid \Box\phi$$

- Syntactic sugar:

  $$\neg\phi \equiv \phi \to \bot \qquad \top \equiv \neg\bot$$

  $$\phi \vee \psi \equiv \neg\phi \to \psi \qquad \phi \wedge \psi \equiv \neg(\phi \to \neg\psi)$$

  $$[a]\phi \equiv \neg\langle a \rangle\neg\phi \qquad \Box\phi \equiv \neg\Diamond\neg\phi$$

DMBI Logic - Semantics

Semantics

- Resource monoid: $\mathcal{R} = (R, \bullet, e)$
    - R is a set of *resources*
    - $e \in R$ is the unit resource
    - $\bullet : R \times R \to R$ such that, for any $r_1, r_2, r_3 \in R$:
        - Neutral element: $r_1 \bullet e = e \bullet r_1 = r_1$
        - Commutativity: $r_1 \bullet r_2 = r_2 \bullet r_1$
        - Associativity: $r_1 \bullet (r_2 \bullet r_3) = (r_1 \bullet r_2) \bullet r_3$

  Remark: $\bullet$ is total because a resource is viewed as a multiset of atomic resources

DMBI Logic - Semantics

Semantics

- Action monoid (non commutative): $\mathcal{A} = (Act, \odot, 1)$
  - *Act* is a set of *actions*
  - $1 \in Act$ is the unit action
  - $\odot : Act \times Act \to Act$ such that, for any $a_1, a_2, a_3 \in Act$:
    - Neutral element: $a_1 \odot 1 = 1 \odot a_1 = a_1$
    - Associativity: $a_1 \odot (a_2 \odot a_3) = (a_1 \odot a_2) \odot a_3$

Remark: actions are viewed as lists of atomic actions

DMBI Logic - Semantics

Semantics

- A μ-dynamic resource monoid: $\mathcal{M} = (\mathcal{R}, \mathcal{A}, S, \|\cdot\rangle\!\rangle, \mu)$
  - S is a set of *states*
  - $\|\cdot\rangle\!\rangle \subseteq S \times Act \times S$, such that:
    - $\|\cdot\rangle\!\rangle$-unit: $s_1 \,\|1\rangle\!\rangle\, s_1$
    - $\|\cdot\rangle\!\rangle$-composition: if $s_1 \,\|a_1\rangle\!\rangle\, s_2$ and $s_2 \,\|a_2\rangle\!\rangle\, s_3$ then $s_1 \,\|a_1 \odot a_2\rangle\!\rangle\, s_3$
  - $\mu : Act \times R \rightharpoonup R$, such that:
    - μ-unit: $\mu(1, r) \downarrow$ and $\mu(1, r) = r$
    - μ-composition: if $\mu(a_1, r) \downarrow$ and $\mu(a_2, \mu(a_1, r)) \downarrow$ then $\mu(a_1 \odot a_2, r) \downarrow$ and $\mu(a_1 \odot a_2, r) = \mu(a_2, \mu(a_1, r))$
- Denotations:
  - $r, s \xrightarrow{a} r', s'$ iff $\mu(a, r) \downarrow$, $\mu(a, r) = r'$ and $s \,\|a\rangle\!\rangle\, s'$
  - $r, s \rightsquigarrow r', s'$ iff $r, s \xrightarrow{a_0} r_1, s_1 \xrightarrow{a_1} \ldots \xrightarrow{a_{n-1}} r_n, s_n \xrightarrow{a_n} r', s'$

DMBI Logic - Semantics

Semantics

- μ-Model: $\mathcal{K} = (\mathcal{M}, [\![\cdot]\!], |\cdot|, \vDash_{\mathcal{K}})$
  - $r, s \vDash_{\mathcal{K}} p$ iff $(r, s) \in [\![p]\!]$
  - $r, s \vDash_{\mathcal{K}} \bot$ never
  - $r, s \vDash_{\mathcal{K}} \mathrm{I}$ iff $r = e$
  - $r, s \vDash_{\mathcal{K}} \phi \to \psi$ iff $r, s \vDash_{\mathcal{K}} \phi \Rightarrow r, s \vDash_{\mathcal{K}} \psi$
  - $r, s \vDash_{\mathcal{K}} \phi * \psi$ iff $\exists r_1, r_2 \in R \cdot r = r_1 \bullet r_2$ and $r_1, s \vDash_{\mathcal{K}} \phi$ and $r_2, s \vDash_{\mathcal{K}} \psi$
  - $r, s \vDash_{\mathcal{K}} \phi \mathrel{-\!\!*} \psi$ iff $\forall r' \in R \cdot r', s \vDash_{\mathcal{K}} \phi \Rightarrow r \bullet r', s \vDash_{\mathcal{K}} \psi$
  - $r, s \vDash_{\mathcal{K}} \langle a \rangle \phi$ iff $\exists r' \in R \cdot \exists s' \in S \cdot r, s \xrightarrow{|a|} r', s'$ and $r', s' \vDash_{\mathcal{K}} \phi$
  - $r, s \vDash_{\mathcal{K}} \Diamond \phi$ iff $\exists r' \in R \cdot \exists s' \in S \cdot r, s \rightsquigarrow r', s'$ and $r', s' \vDash_{\mathcal{K}} \phi$
- Validity: ϕ is *valid* iff $r, s \vDash_{\mathcal{K}} \phi$ for any $\mathcal{K}$, r and s

Plan

1. Language and semantics
2. Expressiveness
3. Tableaux method
4. Counter-model extraction
5. Conclusions - Perspectives

DMBI Logic - Expressiveness

Concurrent processes modelling

- A user gives a description $\mathcal{D}$ of n concurrent processes $(P_1, ..., P_n)$, where $n \geqslant 1$:

  $\mathcal{D} = (R_{atom}, Act_{atom}, \mu_{pre}, \mu_{post}, \{P_1, ..., P_n\})$, such that:

  - R_{atom} is a set of *atomic resources*
  - Act_{atom} is a set of *atomic actions*
  - $\mu_{pre} : Act_{atom} \to \mathfrak{M}(R_{atom})$
    $\Rightarrow$ $\mu_{pre}(a)$ is the multiset of resources *consumed* when a process performs the action a
  - $\mu_{post} : Act_{atom} \to \mathfrak{M}(R_{atom})$
    $\Rightarrow$ $\mu_{post}(a)$ is the multiset of resources *produced* when a process performs the action a
  - $P_i = (S_i, \twoheadrightarrow_{P_i})$ are processes: S_i is the set of states of P_i and $\twoheadrightarrow_{P_i} \subseteq S_i \times Act_{atom} \times S_i$ is the transition relation of P_i

- ▶ We aim to construct a μ-model that models $\mathcal{D}$

DMBI Logic - Expressiveness

Concurrent processes modelling - Resources

- Denotations for resources:
  - $\mathfrak{M}(R_{atom})$ is the set of all multisets over R_{atom} (functions $R_{atom} \to \mathbb{N}$)
  - e is the empty multisets $(\forall r \in R_{atom} \cdot e(r) = 0)$
  - $R_1 \leq R_2$ iff $R_1(r) \leqslant R_2(r)$ for all $r \in R_{atom}$
  - $R_1 + R_2 = R_3$ such that $R_3(r) = R_1(r) + R_2(r)$ for all $r \in R_{atom}$
  - $R_1 - R_2 = R_3$ such that $R_3(r) = R_1(r) - R_2(r)$ for all $r \in R_{atom}$
    Remark: $R_1 - R_2$ is defined iff $R_2 \leq R_1$.

DMBI Logic - Expressiveness

Concurrent processes modelling - System transitions

- Two denotations for behaviour of the system:
  - $$R \left\{ \begin{array}{ccc} s_1 & \xrightarrow{a_1}_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n & \xrightarrow{a_n}_{P_n} & s_n' \end{array} \right\} R' \quad \text{iff}$$

    $\mu_{pre}(a_1) + \dots + \mu_{pre}(a_n) \leq R$ and
    $R' = R - \mu_{pre}(a_1) - \dots - \mu_{pre}(a_n) + \mu_{post}(a_1) + \dots + \mu_{post}(a_n)$
    and $s_i \xrightarrow{a_i}_{P_i} s_i'$ for all $i \in \{1, \dots, n\}$.
  - $$R \left\{ \begin{array}{ccc} s_1 & \dashrightarrow_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n & \dashrightarrow_{P_n} & s_n' \end{array} \right\} R' \quad \text{iff}$$

    $$R \left\{ \begin{array}{ccc} s_1 & \xrightarrow{a_1^1}_{P_1} & s_1^1 \\ \vdots & & \vdots \\ s_n & \xrightarrow{a_n^1}_{P_n} & s_n^1 \end{array} \right\} R_1 \quad \dots \quad R_{k-1} \left\{ \begin{array}{ccc} s_1^{k-1} & \xrightarrow{a_1^k}_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n^{k-1} & \xrightarrow{a_n^k}_{P_n} & s_n' \end{array} \right\} R'$$

DMBI Logic - Expressiveness

Concurrent processes modelling - Synchronous/Asynchronous

- Synchronous VS asynchronous processes:
  - At each transition all processes perform an action:

    ⇒ synchronous processes
  - How to model asynchronous processes?
    - By considering an atomic action *skip*
    - $\mu_{pre}(skip) = \mu_{post}(skip) = e$
    - $s_i \xrightarrow{skip}_{P_i} s_i$ for all processes P_i and all states $s_i \in S_i$
  - Example: $R \left\{ \begin{array}{lll} s_1 & \xrightarrow{a_1}_{P_1} & s_1' \\ s_2 & \xrightarrow{skip}_{P_2} & s_2 \\ s_3 & \xrightarrow{a_3}_{P_3} & s_3' \end{array} \right\} R'$

    ⇒ only P_1 and P_3 perform an action in this step

DMBI Logic - Expressiveness

Concurrent processes modelling - Actions

- Denotations for actions:
  - We define $Act^{\#}_{atom} = \{a_1\#...\#a_n \mid a_1, ..., a_n \in Act_{atom}\}$

    $\Rightarrow$ $a_1\#a_2$ is a *concurrent atomic action* where P_1 performs a_1 and P_2 performs a_2.

  - We define $\mathfrak{L}(Act^{\#}_{atom})$ the set of all lists built on $Act^{\#}_{atom}$:

    For example $[a_1\#a_2\#a_3; a'_1\#a'_2\#a'_3]$ is an action that consists to perform $a_1\#a_2\#a_3$ and then $a'_1\#a'_2\#a'_3$

  - $[]$ is the empty list

  - $\oplus$ is the concatenation of lists

Propositions

1. $\mathcal{R} = (\mathfrak{M}(R_{atom}), +, e)$ is a resource monoid.
2. $\mathcal{A} = (\mathfrak{L}(Act^{\#}_{atom}), \oplus, [])$ is an action monoid

DMBI Logic - Expressiveness

Concurrent processes modelling - States and the μ function

- Denotations for states:
  - $S^{\#} = \{s_1\#...\#s_n \mid s_i \in S_i \text{ for any } 1 \leqslant i \leqslant n\}$

    $\Rightarrow$ $s_1\#s_2$ is the state such that P_1 is in state s_1 and P_2 is in state s_2.

- Denotations for the μ function:
  - $\mu^{\#} : Act^{\#}_{atom} \times \mathfrak{M}(R_{atom}) \rightharpoonup \mathfrak{M}(R_{atom})$

$$\mu^{\#}(a_1\#...\#a_n, R) = \begin{cases} \uparrow & \text{if } \mu_{pre}(a_1) + ... + \mu_{pre}(a_n) \not\leq R \\ R - \mu_{pre}(a_1) - ... - \mu_{pre}(a_n) & \\ \quad +\mu_{post}(a_1) + ... + \mu_{post}(a_n) & \text{otherwise} \end{cases}$$

  - $\mu_{list} : \mathfrak{L}(Act^{\#}_{atom}) \times \mathfrak{M}(R_{atom}) \rightharpoonup \mathfrak{M}(R_{atom})$

$$\mu_{list}(L, R) = \begin{cases} R & \text{if } L = [] \\ \uparrow & \text{if } L = [A_1; ...; A_k] \text{ and } \mu^{\#}(A_1, R)\uparrow \\ \mu_{list}([A_2; ...; A_k], \mu^{\#}(A_1, R)) & \text{where } L = [A_1; ...; A_k] \end{cases}$$

DMBI Logic - Expressiveness

Concurrent processes modelling - State/resource relation

- Denotations for the relation on states and resources:
  - $|\cdot\rangle^{\#} : S^{\#} \times Act^{\#}_{atom} \times S^{\#}$

$$s_1\#...\#s_n \, |a_1\#...\#a_n\rangle^{\#} \, s'_1\#...\#s'_n \text{ iff } s_i \xrightarrow{a_i}_{P_i} s'_i \text{ for all } 1 \leqslant i \leqslant n$$

  - $|\cdot\rangle_{list} : S^{\#} \times \mathfrak{L}(Act^{\#}_{atom}) \times S^{\#}$

$$S \, |[A_1; ...; A_k]\rangle_{list} \, S' \text{ iff } S \, |A_1\rangle^{\#} \, S_1 \, |A_2\rangle^{\#} \, ... \, |A_{k-1}\rangle^{\#} \, S_{k-1} \, |A_k\rangle^{\#} \, S'$$

Lemma

Let $\mathcal{D} = (R_{atom}, Act_{atom}, \mu_{pre}, \mu_{post}, \{P_1, ..., P_n\})$, where $P_i = (S_i, \rightarrow_{P_i})$.
$\mathcal{M} = (\mathcal{R}, \mathcal{A}, S^{\#}, |\cdot\rangle_{list}, \mu_{list})$, where $\mathcal{R} = (\mathfrak{M}(R_{atom}), +, e)$ and where $\mathcal{A} = (\mathfrak{L}(Act^{\#}_{atom}), \oplus, [])$ is a μ-DRM.

DMBI Logic - Expressiveness

Concurrent processes modelling - Reachability/Satisfiability

- Denotations:
  - $[\![\cdot]\!] : Prop \rightarrow \mathbb{P}(\mathfrak{M}(R_{atom}) \times S^{\#})$

$$[\![r_i]\!] = \{(\{r_i\}, s) \mid s \in S^{\#}\}$$

  - $|\cdot| : S_{Act} \rightarrow \mathfrak{L}(Act^{\#}_{atom})$

$$|a_1\#...\#a_n| = [a_1\#...\#a_n]$$

  - $\widehat{\cdot} : \mathfrak{M}(R_{atom}) \rightarrow \mathcal{L}$:

$$\widehat{R} = \begin{cases} \mathrm{I} & \text{if } R = e \\ r_1 * ... * r_k & \text{if } R = \{r_1, ..., r_k\} \end{cases}$$

DMBI Logic - Expressiveness

Concurrent processes modelling - Reachability/Satisfiability

Lemma

$$R\left\{\begin{array}{ccc} s_1 & \overset{a_1}{\longrightarrow}_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n & \overset{a_n}{\longrightarrow}_{P_n} & s_n' \end{array}\right\} R' \quad \text{iff} \quad R, s_1\#\ldots\#s_n \xrightarrow{[a_1\#\ldots\#a_n]} R', s_1'\#\ldots\#s_n'$$

Lemma

$$R\left\{\begin{array}{ccc} s_1 & \dashrightarrow_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n & \dashrightarrow_{P_n} & s_n' \end{array}\right\} R' \quad \text{iff} \quad R, s_1\#\ldots\#s_n \rightsquigarrow R', s_1'\#\ldots\#s_n'$$

DMBI Logic - Expressiveness

Concurrent processes modelling - Reachability/Satisfiability

Theorem

$$R \left\{ \begin{array}{ccc} s_1 & \xrightarrow{a_1}_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n & \xrightarrow{a_n}_{P_n} & s_n' \end{array} \right\} R' \quad \text{iff} \quad R, s_1 \# \ldots \# s_n \vDash_{\mathcal{K}} \langle a_1 \# \ldots \# a_n \rangle \widehat{R'}$$

Theorem

$$R \left\{ \begin{array}{ccc} s_1 & \dashrightarrow_{P_1} & s_1' \\ \vdots & & \vdots \\ s_n & \dashrightarrow_{P_n} & s_n' \end{array} \right\} R' \quad \text{iff} \quad R, s_1 \# \ldots \# s_n \vDash_{\mathcal{K}} \Diamond \widehat{R'}$$

DMBI Logic - Expressiveness

Concurrent processes modelling - Mutual exclusion

- Mutual exclusion (revisited):
  $\mathcal{D} = (R_{atom}, Act_{atom}, \mu_{pre}, \mu_{post}, \{P_1, P_2\})$, where:
  - $R_{atom} = \{J\}$
  - $Act_{atom} = \{a_{nc}, a_c, a_p, a_v\}$
  - μ_{pre} is defined by:
    - $\mu_{pre}(a_{nc}) = \mu_{pre}(a_c) = \mu_{pre}(a_v) = e$
    - $\mu_{pre}(a_p) = J$
  - μ_{post} is defined by:
    - $\mu_{post}(a_{nc}) = \mu_{post}(a_c) = \mu_{post}(a_p) = e$
    - $\mu_{post}(a_v) = J$
  - $P_1 = (S_1, \rightarrowtriangle_{P_1})$ and $P_2 = (S_2, \rightarrowtriangle_{P_2})$ such that:
    - $S_1 = S_2 = \{s_{nc}, s_c\}$
    - For any $i \in \{1, 2\}$, we have:

$$s_{nc} \overset{a_{nc}}{\rightarrowtriangle}_{P_i} s_{nc} \qquad s_{nc} \overset{a_p}{\rightarrowtriangle}_{P_i} s_c \qquad s_c \overset{a_c}{\rightarrowtriangle}_{P_i} s_c \qquad s_c \overset{a_v}{\rightarrowtriangle}_{P_i} s_{nc}$$

DMBI Logic - Expressiveness

Concurrent processes modelling - Mutual exclusion

- Mutual exclusion (revisited):
  - We construct $\mathcal{M} = (\mathcal{R}, \mathcal{A}, S^{\#}, |\cdot\rangle_{list}, \mu_{list})$, where $\mathcal{R} = (\mathfrak{M}(R_{atom}), +, e)$ and where $\mathcal{A} = (\mathfrak{L}(Act^{\#}_{atom}), \oplus, [])$
  - "After performing any succession of actions, the processes can not perform together a critical action":

$$\{J\}, s_{nc}\#s_{nc} \vDash_{\mathcal{K}} \Box[a_c\#a_c]\bot$$

  - "It is impossible to reach a state such that more than one token is available":

$$\{J\}, s_{nc}\#s_{nc} \vDash_{\mathcal{K}} \neg\Diamond(J * J * \top)$$

- We observe that **DMBI**:
  - captures resource transformation ($\not\approx$ **DBI**) ($\approx$ **MBI**)
  - expresses properties on any reachable states ($\approx$ **DBI**) ($\not\approx$ **MBI**)
  - does not model capture process production ($\not\approx$ **MBI**)

Plan

1 Language and semantics

2 Expressiveness

3 Tableaux method

4 Counter-model extraction

5 Conclusions - Perspectives

DMBI Proof theory - Tableaux method

An extension of **BI** calculus (Galmiche-Méry-Pym 2005) based on constrained set of statements (CSS in Larchey 2012)

- Resource labels (R), action labels (Act) and state labels (S)
- Resource constraints ($=$), μ-constraints (μ) and transition constraints ($\|.\rangle\!\rangle$)
- Signed formulae: $\mathbb{S}\phi : (x, u)$
- Branches are denoted $\langle \mathcal{F}, \mathcal{C} \rangle$ where $\mathcal{C}$ is a set of resource, transition and μ constraints
- Assertions/requirements

DMBI Proof theory - Tableaux method

Labels

- **Resource labels** (L_r):

  $$X ::= 1_r \mid c_i \mid X \circ X$$

  where $c_i \in \gamma_r = \{c_1, c_2, ...\}$ and $\circ$ is a function on L_r that is associative, commutative and 1_r is its unit. $x \circ y$ is denoted xy.

- **Action labels** (L_a):

  $$X ::= 1_a \mid a_i \mid d_i \mid X \centerdot X$$

  where $a_i \in S_{Act}$, $d_i \in \gamma_a = \{d_1, d_2, ...\}$, $S_{Act} \cap \gamma_a = \emptyset$ and $\centerdot$ is a function on L_a that is associative (not commutative) and 1_a is its unit. $f \centerdot g$ is denoted fg.

- **State labels** (L_s): $L_s = \{l_1, l_2, ...\}$.

DMBI Proof theory - Tableaux method

Constraints

- Resource constraints:
  - encode equality on resources.
  - $x \sim y$ where x and y are resource labels.
- μ-constraints:
  - encode the function μ.
  - $x \overset{f}{\twoheadrightarrow} y$ where x and y are resource labels and f is an action label.
- Transition constraints:
  - Encode the function $\|\cdot\rangle\!\rangle$.
  - $u \overset{f}{\rightarrowtail} v$ where u and v are state labels and f is an action label.

DMBI Proof theory - Tableaux method

Constraint closure

- Rules that product resource constraints:

$$\frac{}{1_r \sim 1_r}\ \langle 1_r \rangle \qquad \frac{x \sim y}{y \sim x}\ \langle s_r \rangle \qquad \frac{xy \sim xy}{x \sim x}\ \langle d_r \rangle$$

$$\frac{x \sim y \qquad y \sim z}{x \sim z}\ \langle t_r \rangle \qquad \frac{x \sim x' \qquad y \sim y'}{xy \sim x'y'}\ \langle g_r \rangle$$

$$\frac{x \overset{f}{\twoheadrightarrow} y \qquad x \overset{f}{\twoheadrightarrow} z}{y \sim z}\ \langle k_r \rangle \qquad \frac{x \overset{f}{\twoheadrightarrow} y}{x \sim x}\ \langle a_{r_1} \rangle$$

DMBI Proof theory - Tableaux method

Constraint closure

- Rules that product μ-constraints:

$$\frac{x \sim x}{x \overset{1_a}{\twoheadrightarrow} x}\ \langle 1_\mu \rangle \qquad \frac{x \overset{f}{\twoheadrightarrow} y \qquad y \overset{g}{\twoheadrightarrow} z}{x \overset{fg}{\twoheadrightarrow} z}\ \langle t_\mu \rangle$$

$$\frac{x \overset{f}{\twoheadrightarrow} y \qquad x \sim x'}{x' \overset{f}{\twoheadrightarrow} y}\ \langle k_{\mu_1} \rangle \qquad \frac{x \overset{f}{\twoheadrightarrow} y \qquad y \sim y'}{x \overset{f}{\twoheadrightarrow} y'}\ \langle k_{\mu_2} \rangle$$

- Rules that product transition constraints:

$$\frac{u \overset{f}{\rightarrowtail} v}{u \overset{1_a}{\rightarrowtail} u}\ \langle 1_{t_1} \rangle \qquad \frac{u \overset{f}{\rightarrowtail} v}{v \overset{1_a}{\rightarrowtail} v}\ \langle 1_{t_2} \rangle \qquad \frac{u \overset{f}{\rightarrowtail} v \qquad v \overset{g}{\rightarrowtail} w}{u \overset{fg}{\rightarrowtail} w}\ \langle t_t \rangle$$

DMBI Tableaux method

Modal rules

- **Assertion** rules:
  Introduction of new labels and assertions (or constraints)

$$\frac{\mathbb{T}\langle f\rangle\phi : (x,u) \in \mathcal{F}}{\langle\{\mathbb{T}\phi : (c_i, l_i)\}, \{x \overset{f}{\twoheadrightarrow} c_i, u \overset{f}{\rightarrowtail} l_i\}\rangle}\ \langle\mathbb{T}\langle - \rangle\rangle$$

$$\frac{\mathbb{T}\Diamond\phi : (x,u) \in \mathcal{F}}{\langle\{\mathbb{T}\phi : (c_i, l_i)\}, \{x \overset{d_i}{\twoheadrightarrow} c_i, u \overset{d_i}{\rightarrowtail} l_i\}\rangle}\ \langle\mathbb{T}\Diamond\rangle$$

- **Requirement** rules:
  Conditions that must be verified in the closure of constraints

$$\frac{\mathbb{F}\langle f\rangle\phi : (x,u) \in \mathcal{F} \text{ and } x \overset{f}{\twoheadrightarrow} y \in \overline{\mathcal{C}} \text{ and } u \overset{f}{\rightarrowtail} v \in \overline{\mathcal{C}}}{\langle\mathbb{F}\phi : (y,v), \emptyset\rangle}\ \langle\mathbb{F}\langle - \rangle\rangle$$

$$\frac{\mathbb{F}\Diamond\phi : (x,u) \in \mathcal{F} \text{ and } x \overset{f}{\twoheadrightarrow} y \in \overline{\mathcal{C}} \text{ and } u \overset{f}{\rightarrowtail} v \in \overline{\mathcal{C}}}{\langle\{\mathbb{F}\phi : (y,v)\}, \emptyset\rangle}\ \langle\mathbb{F}\Diamond\rangle$$

DMBI Tableaux method

Definition: closed branch

A CSS (branch) $\langle \mathcal{F}, \mathcal{C} \rangle$ is **closed** iff one of these conditions holds:

- $\mathbb{T}\phi : (x, u) \in \mathcal{F}$, $\mathbb{F}\phi : (y, u) \in \mathcal{F}$ and $x \sim y \in \overline{\mathcal{C}}$
- $\mathbb{F}\mathrm{I} : (x, u) \in \mathcal{F}$ and $1_r \sim x \in \overline{\mathcal{C}}$
- $\mathbb{T}\bot : (x, u) \in \mathcal{F}$

Definition: μ-proof

A *μ-proof* for a formula ϕ is a μ-tableau for ϕ which is closed.

Theorem: soundness

If there exists a μ-proof for a formula ϕ then ϕ is valid.

Theorem: completeness

If a formula ϕ is valid then there is a μ-proof for ϕ.

DMBI Tableaux method - an example

- How to prove $\phi \equiv (\mathrm{I} \mathrel{-\!\!*} \langle a \rangle \langle b \rangle P) \rightarrow \Diamond P$?

Step 1: Initialization

$[\mathcal{F}]$

$\mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a \rangle \langle b \rangle P) \rightarrow \Diamond P : (c_1, l_1)$

$[\mathcal{C}]$

$c_1 \sim c_1 \qquad l_1 \overset{1_a}{\rightarrowtail} l_1$

DMBI Tableaux method - an example

$$[\mathcal{F}]$$

$$\mathbb{F}(\mathrm{I} \multimap \langle a\rangle\langle b\rangle P) \to \Diamond P : (c_1, l_1)$$

$$[\mathcal{C}]$$

$$c_1 \sim c_1 \qquad l_1 \overset{1_a}{\rightarrowtail} l_1$$

DMBI Tableaux method - an example

$$[\mathcal{F}]$$

$$\surd_1 \ \mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \to \Diamond P : (c_1, l_1)$$

$$|$$

$$\mathbb{T}\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P : (c_1, l_1)$$

$$\mathbb{F}\Diamond P : (c_1, l_1)$$

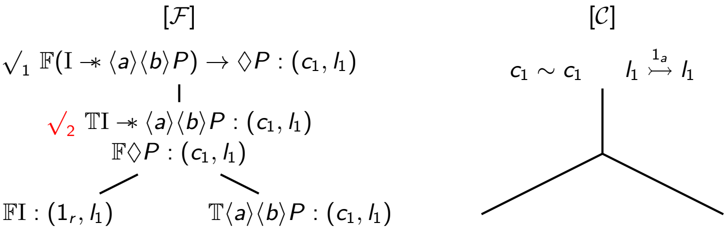
$$[\mathcal{C}]$$

$$c_1 \sim c_1 \qquad l_1 \overset{1_a}{\rightarrowtail} l_1$$

$$|$$

$$\frac{\mathbb{F}\phi \to \psi : (x, u) \in \mathcal{F}}{\langle \{\mathbb{T}\phi : (x, u), \mathbb{F}\psi : (x, u)\}, \emptyset\rangle} \ \langle \mathbb{F} \to \rangle$$

DMBI Tableaux method - an example



$[\mathcal{F}]$

$\surd_1\ \mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \to \Diamond P : (c_1, l_1)$

$\surd_2\ \mathbb{T}\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\mathbb{F}\Diamond P : (c_1, l_1)$

$\mathbb{F}\mathrm{I} : (1_r, l_1)$ | $\mathbb{T}\langle a\rangle\langle b\rangle P : (c_1, l_1)$

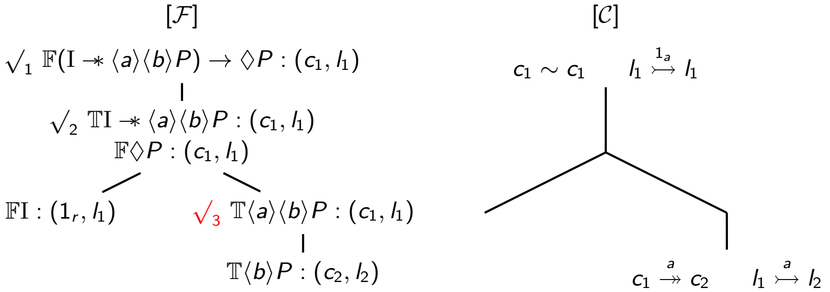
$[\mathcal{C}]$

$c_1 \sim c_1 \qquad l_1 \overset{1_a}{\rightarrowtail} l_1$

$$\frac{\mathbb{T}\phi \mathrel{-\!\!*} \psi : (x, u) \in \mathcal{F} \text{ and } xy \sim xy \in \overline{\mathcal{C}}}{\langle\{\mathbb{F}\phi : (y, u)\}, \emptyset\rangle \ \mid\ \langle\{\mathbb{T}\psi : (xy, u)\}, \emptyset\rangle}\ \langle\mathbb{T}\mathrel{-\!\!*}\rangle$$

Remark: $c_1 \circ 1_r = c_1$

DMBI Tableaux method - an example



$[\mathcal{F}]$

$\surd_1\ \mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \rightarrow \Diamond P : (c_1, l_1)$

$\surd_2\ \mathbb{T}\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\mathbb{F}\Diamond P : (c_1, l_1)$

$\mathbb{F}\mathrm{I} : (1_r, l_1)$

$\surd_3\ \mathbb{T}\langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\mathbb{T}\langle b\rangle P : (c_2, l_2)$

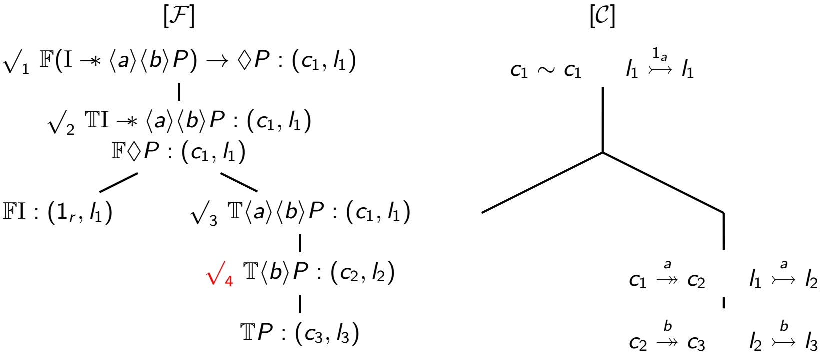
$[\mathcal{C}]$

$c_1 \sim c_1 \qquad l_1 \overset{1_a}{\rightarrowtail} l_1$

$c_1 \overset{a}{\twoheadrightarrow} c_2 \qquad l_1 \overset{a}{\rightarrowtail} l_2$

$$\frac{\mathbb{T}\langle f\rangle\phi : (x, u) \in \mathcal{F}}{\langle\{\mathbb{T}\phi : (c_i, l_i)\}, \{x \overset{f}{\twoheadrightarrow} c_i, u \overset{f}{\rightarrowtail} l_i\}\rangle}\ \langle\mathbb{T}\langle-\rangle\rangle$$

DMBI Tableaux method - an example



$[\mathcal{F}]$

$\surd_1\ \mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \to \Diamond P : (c_1, l_1)$

$\surd_2\ \mathbb{T}\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\mathbb{F}\Diamond P : (c_1, l_1)$

$\mathbb{F}\mathrm{I} : (1_r, l_1)$

$\surd_3\ \mathbb{T}\langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\surd_4\ \mathbb{T}\langle b\rangle P : (c_2, l_2)$

$\mathbb{T}P : (c_3, l_3)$

$[\mathcal{C}]$

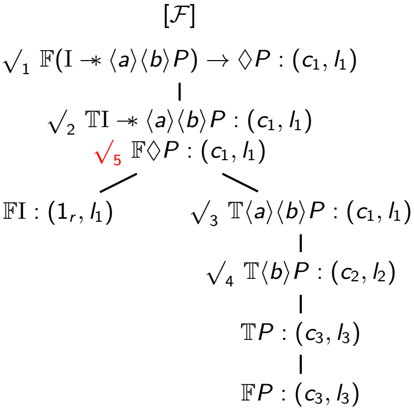
$c_1 \sim c_1 \qquad l_1 \xrightarrow{1_a} l_1$

$c_1 \overset{a}{\twoheadrightarrow} c_2 \qquad l_1 \overset{a}{\rightarrowtail} l_2$

$c_2 \overset{b}{\twoheadrightarrow} c_3 \qquad l_2 \overset{b}{\rightarrowtail} l_3$

$$\frac{\mathbb{T}\langle f\rangle\phi : (x, u) \in \mathcal{F}}{\langle\{\mathbb{T}\phi : (c_i, l_i)\}, \{x \overset{f}{\twoheadrightarrow} c_i, u \overset{f}{\rightarrowtail} l_i\}\rangle}\ \langle\mathbb{T}\langle-\rangle\rangle$$

DMBI Tableaux method - an example



$[\mathcal{F}]$

$\surd_1\ \mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \to \Diamond P : (c_1, l_1)$

$\surd_2\ \mathbb{T}\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\surd_5\ \mathbb{F}\Diamond P : (c_1, l_1)$

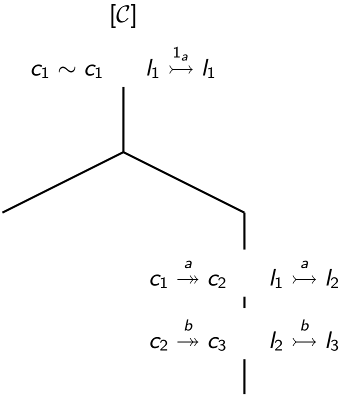
$\mathbb{F}\mathrm{I} : (1_r, l_1)$

$\surd_3\ \mathbb{T}\langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\surd_4\ \mathbb{T}\langle b\rangle P : (c_2, l_2)$

$\mathbb{T}P : (c_3, l_3)$

$\mathbb{F}P : (c_3, l_3)$



$[\mathcal{C}]$

$c_1 \sim c_1 \quad l_1 \overset{1_a}{\rightarrowtail} l_1$

$c_1 \overset{a}{\twoheadrightarrow} c_2 \quad l_1 \overset{a}{\rightarrowtail} l_2$

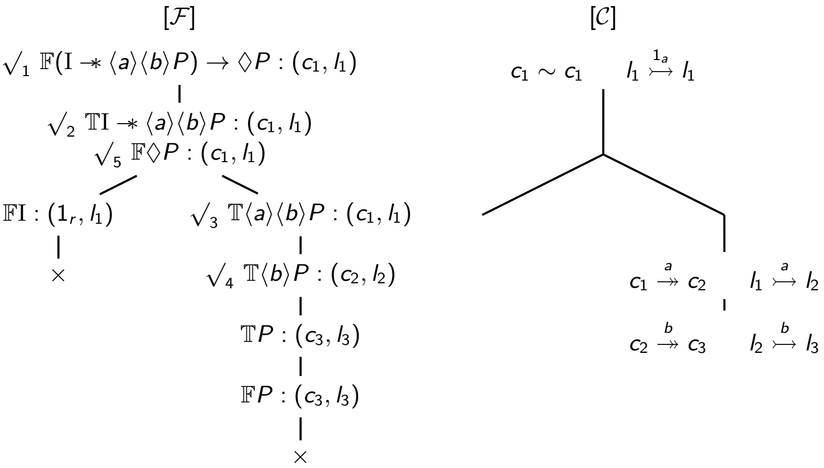
$c_2 \overset{b}{\twoheadrightarrow} c_3 \quad l_2 \overset{b}{\rightarrowtail} l_3$

$$\frac{\mathbb{F}\Diamond\phi : (x, u) \in \mathcal{F} \text{ and } x \overset{f}{\twoheadrightarrow} y \in \overline{\mathcal{C}} \text{ and } u \overset{f}{\rightarrowtail} v \in \overline{\mathcal{C}}}{\langle\{\mathbb{F}\phi : (y, v)\}, \emptyset\rangle}\ \langle\mathbb{F}\Diamond\rangle$$

$$\frac{c_1 \overset{a}{\twoheadrightarrow} c_2 \qquad c_2 \overset{b}{\twoheadrightarrow} c_3}{c_1 \overset{ab}{\twoheadrightarrow} c_3}\ \langle t_\mu\rangle \qquad \frac{l_1 \overset{a}{\rightarrowtail} l_2 \qquad l_2 \overset{b}{\rightarrowtail} l_3}{l_1 \overset{ab}{\rightarrowtail} l_3}\ \langle t_t\rangle$$

DMBI Tableaux method - an example

Step 2: Application of rules



$[\mathcal{F}]$

$\surd_1\ \mathbb{F}(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \rightarrow \Diamond P : (c_1, l_1)$

$\surd_2\ \mathbb{T}\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\surd_5\ \mathbb{F}\Diamond P : (c_1, l_1)$

$\mathbb{F}\mathrm{I} : (1_r, l_1)$

$\times$

$\surd_3\ \mathbb{T}\langle a\rangle\langle b\rangle P : (c_1, l_1)$

$\surd_4\ \mathbb{T}\langle b\rangle P : (c_2, l_2)$

$\mathbb{T}P : (c_3, l_3)$

$\mathbb{F}P : (c_3, l_3)$

$\times$

$[\mathcal{C}]$

$c_1 \sim c_1 \qquad l_1 \overset{1_a}{\rightarrowtail} l_1$

$c_1 \overset{a}{\twoheadrightarrow} c_2 \qquad l_1 \overset{a}{\rightarrowtail} l_2$

$c_2 \overset{b}{\twoheadrightarrow} c_3 \qquad l_2 \overset{b}{\rightarrowtail} l_3$

The formula $(\mathrm{I} \mathrel{-\!\!*} \langle a\rangle\langle b\rangle P) \rightarrow \Diamond P$ is valid

Plan

1. Language and semantics
2. Expressiveness
3. Tableaux method
4. **Counter-model extraction**
5. Conclusions - Perspectives

DMBI Counter-model extraction

Counter-model extraction

Definition: Hintikka CSS

A Hintikka CSS $\langle \mathcal{F}, \mathcal{C} \rangle$ is a unclosed branch such that "all information has been extracted":

1 $\mathbb{T}\phi : (x, u) \notin \mathcal{F}$ or $\mathbb{F}\phi : (y, u) \notin \mathcal{F}$ or $x \sim y \notin \overline{\mathcal{C}}$

2-12 ...

13 If $\mathbb{T}\Diamond\phi : (x, u) \in \mathcal{F}$ then $\exists y \in L_r$, $\exists f \in L_a$, $\exists v \in L_s$, $x \overset{f}{\twoheadrightarrow} y \in \overline{\mathcal{C}}$ and $u \overset{f}{\rightarrowtail} v \in \overline{\mathcal{C}}$ and $\mathbb{T}\phi : (y, v) \in \mathcal{F}$

14 If $\mathbb{F}\Diamond\phi : (x, u) \in \mathcal{F}$ then $\forall y \in L_r$, $\forall f \in L_a$, $\forall v \in L_s$, $(x \overset{f}{\twoheadrightarrow} y \in \overline{\mathcal{C}}$ and $u \overset{f}{\rightarrowtail} v \in \overline{\mathcal{C}}) \Rightarrow \mathbb{F}\phi : (y, v) \in \mathcal{F}$

Lemma: counter-model extraction

A counter-model can be extracted from a Hintikka branch.

DMBI Counter-model extraction

Counter-model extraction

Function Ω

Let $\langle \mathcal{F}, \mathcal{C} \rangle$ be a Hintikka CSS. $\Omega(\langle \mathcal{F}, \mathcal{C} \rangle) = (\mathcal{M}, [\![\cdot]\!], |\cdot|, \vDash_{\mathcal{K}})$, such that:

- $R = \mathcal{D}_r(\overline{\mathcal{C}})/\sim \qquad S = \mathcal{A}_s(\mathcal{C}) \qquad Act = \mathcal{D}_a(\overline{\mathcal{C}}) \cup \{\alpha\}$ (where $\alpha \notin \mathcal{D}_a(\overline{\mathcal{C}})$)
- $e = [1_r]$
- $1 = 1_a$
- $[x] \bullet [y] = [x \circ y]$
- $\mu(a, [x]) = \begin{cases} \uparrow & \text{if } \{y \mid x \overset{a}{\twoheadrightarrow} y \in \overline{\mathcal{C}}\} = \emptyset \\ \{y \mid x \overset{a}{\twoheadrightarrow} y \in \overline{\mathcal{C}}\} & \text{otherwise} \end{cases}$
- $s_1 \,\| f \rangle\!\rangle\, s_2$ iff $s_1 \overset{f}{\rightarrowtail} s_2 \in \overline{\mathcal{C}}$
- For all $a_1, a_2 \in Act$, $a_1 \odot a_2 = \begin{cases} a_1 \cdot a_2 & \text{if } a_1 \cdot a_2 \in \mathcal{D}_a(\overline{\mathcal{C}}) \\ \alpha & \text{otherwise} \end{cases}$
- For all $a \in S_{Act}$, $|a| = \begin{cases} a & \text{if } a \in \mathcal{D}_a(\overline{\mathcal{C}}) \\ \alpha & \text{otherwise} \end{cases}$
- $([x], s) \in [\![P]\!]$ iff $\exists y \in L_r, x \in [y]$ and $\mathbb{T}P : (y, s) \in \mathcal{F}$

Plan

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Conclusions

Conclusions

A modal extension of **BBI** for resource transformations

- That captures resource transformations ($\approx$ **MBI**)
- That includes modalities $\Diamond$ and $\Box$ ($\approx$ **DBI**)
- That has a sound and complete calculus with a countermodel extraction method
- That can express properties resources produced by *n* concurrent processes that manipulate these resources

Future works

Future works

- Study is **DMBI** can capture process production
- Study extension of **DMBI** with locations and provide a sound and complete calculus
- Study other extension to express properties on:
    - Webservices
    - Protocols
    - ...