Dynamic Epistemic Logics with Quantification

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Public Announcement Logic

Syntax

- Proposed by [Plaza, ISMIS 1989]
- Vocabulary:
  - a countable set $P = \{p, q, \ldots\}$ of propositional variables
  - a finite set $N = \{i, j, \ldots\}$ of labels denoting agents
- Language $\mathcal{L}_{\mathrm{PAL}}$:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{K}_i\varphi \mid [\varphi]\varphi$$

  where p ranges over P and i over N
- Common abbreviations for $\bot$, $\vee$, $\rightarrow$ and $\leftrightarrow$ plus:

$$\langle\psi\rangle\varphi \overset{\text{def}}{=} \neg[\psi]\neg\varphi$$

- The language of *epistemic logic* $\mathcal{L}_{\mathrm{EL}}$ is $\mathcal{L}_{\mathrm{PAL}}$ without the operator [ ].

Public Announcement Logic

Intended Meanings

- $K_i\varphi$: 'agent i knows that φ is true'
- $[\psi]\varphi$: 'after the truthful public announcement ψ, φ is true'
- $\langle\psi\rangle\varphi$: '$\psi$ can be truthfully announced and φ is true after it'

Public Announcement Logic

Epistemic Models and Updates

- An *epistemic model* is a triple $M = \langle W, R, V \rangle$ where:
  - W is a non-empty set of possible worlds
  - $R : N \to (W \times W)$, where:
    for each $i \in N$, it returns an equivalence relation on W
  - $V : P \to 2^W$
- The *update* of M by the public announcement φ is the triple $M|\varphi = \langle W|\varphi, R|\varphi, V|\varphi \rangle$, where:

$$W|\varphi = \{w : M, w \models \varphi\}$$

$$R|\varphi(i) = R(i) \cap (W|\varphi \times W|\varphi)$$

$$V|\varphi(p) = V(p) \cap W|\varphi$$

Public Announcement Logic

Semantics

$$
\begin{array}{lll}
M, w \models \top & & \\
M, w \models p & \text{iff} & w \in V(p) \\
M, w \models \neg\varphi & \text{iff} & M, w \not\models \varphi \\
M, w \models \varphi \wedge \psi & \text{iff} & M, w \models \varphi \text{ and } M, w \models \psi \\
M, w \models \mathrm{K}_i\varphi & \text{iff} & \text{for all } v \in W, \text{ if } (w, v) \in R(i) \text{ then } M, v \models \varphi \\
 & & \\
M, w \models [\varphi]\psi & \text{iff} & M, w \models \varphi \text{ implies } M|\varphi, w \models \psi
\end{array}
$$

- Validity and satisfiability are defined as usual.

Public Announcement Logic

Example

- Moore sentence:
  'p is true and you do not know it'
- The Moore sentence is not (always) successful:
  $\not\models [p \wedge \neg K_i p](p \wedge \neg K_i p)$
- Because the public announcement $p \wedge \neg K_i p$ deletes the worlds where p is false.
- This is called "knowability paradox". PAL can be used to model these sentences.

Public Announcement Logic

Axiomatization

- All principles for EL
- Reduction axioms:

$$[\psi]p \leftrightarrow (\psi \rightarrow p)$$
$$[\psi]\neg\varphi \leftrightarrow (\psi \rightarrow \neg[\psi]\varphi)$$
$$[\psi](\varphi_1 \wedge \varphi_2) \leftrightarrow ([\psi]\varphi_1 \wedge [\psi]\varphi_2)$$
$$[\psi]\mathrm{K}_i\varphi \leftrightarrow (\psi \rightarrow \mathrm{K}_i[\psi]\varphi)$$

Public Announcement Logic

Some Interesting Properties

- PAL is decidable:
  - Reduction axioms plus EL decidability [Kooi, JANCL 2007]
- PAL is more succinct than EL [Lutz, AAMAS 2006].
- Satisfiability checking in PAL is PSPACE-Complete:
  - Efficient reduction [Lutz, AAMAS 2006]
  - Optimal tableaux [Balbiani et al., JLC 2010]

Arbitrary Public Announcement Logic

Syntax and Meaning

- Proposed by [Balbiani et al., RSL 2008]
- Language $\mathcal{L}_{\text{APAL}}$:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{K}_i\varphi \mid [\varphi]\varphi \mid \Box\varphi$$

- Abbreviation: $\Diamond\varphi \stackrel{\text{def}}{=} \neg\Box\neg\varphi$.
- Intended meanings:
  - $\Box\varphi$: ‘φ is true after every truthful public announcement’
  - $\Diamond\varphi$: ‘φ is true after some truthful public announcement’

Arbitrary Public Announcement Logic

Semantics

- Interpretation:

$$M, w \models \Box\varphi \text{ iff } \text{ for all } \psi \in \mathcal{L}_{\mathrm{EL}}, M, w \models [\psi]\varphi$$

- Quantification over $\mathcal{L}_{\mathrm{EL}}$ to avoid a cyclic definition of $\models$
- The logic can model questions like:
  - 'is there a way to make agent i knows that/whether φ?'
  - 'is there a way to make agent i knows that φ while j does not know it?'
  - 'is there a way to make φ become common knowledge?'

Arbitrary Public Announcement Logic

Axiomatization 1

- All principles for PAL
- $\Box\varphi \to [\psi]\varphi$
- From $\eta([\psi]\varphi)$, for all $\psi \in \mathcal{L}_{EL}$, infer $\eta(\Box\varphi)$ where η is a necessity form:

$$\eta ::= \sharp \mid \varphi \to \eta \mid K_i\eta \mid [\varphi]\eta$$

and $\eta(\varphi)$ is obtained from η by substituting $\sharp$ in η by φ.

Arbitrary Public Announcement Logic

Axiomatization 2

- All principles for PAL
- $\Box\varphi \rightarrow [\psi]\varphi$
- From $\varphi \rightarrow [\chi][p]\psi$ infer $\varphi \rightarrow [\chi]\Box\psi$,
  where p does not occur in φ, ψ or χ

Arbitrary Public Announcement Logic

Some Logical Properties

1. $\vdash \Box(\varphi \wedge \psi) \leftrightarrow (\Box\varphi \wedge \Box\psi)$ (K)
2. $\vdash \Box\varphi \rightarrow \varphi$ (T)
3. $\vdash \Box\varphi \rightarrow \Box\Box\varphi$ (4)
4. $\vdash \Box\Diamond\varphi \rightarrow \Diamond\Box\varphi$ (MK)
5. $\vdash \Diamond\Box\varphi \rightarrow \Box\Diamond\varphi$ (CR – Confluence)
6. $\vdash \varphi$ implies $\vdash \Box\varphi$ (Necessitation)

Arbitrary Public Announcement Logic

Other Properties

- Single-agent APAL is not more expressive than EL.
- Multi-agents APAL is more expressive than PAL (and thus, also EL).
- Model checking in APAL is decidable.
- Satisfiability checking in APAL is not decidable [van Ditmarsch and French, AiML 2008].

Group Announcement Logic

Syntax and Semantics

- Proposed by [Ågotnes et al., JAL 2010]
- Language:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{K}_i\varphi \mid [\varphi]\varphi \mid [G]\varphi$$

  where G ranges over 2^N.
- Abbreviation: $\langle G\rangle\varphi \overset{\text{def}}{=} \neg[G]\neg\varphi$
- Intended meanings:
  - $[G]\varphi$: 'φ is true after all truthful public announcements made by group G'
  - $\langle G\rangle\varphi$: φ is true after some truthful public announcement made by group G'

Group Announcement Logic

Semantics

- Interpretation:

$$M, w \models [G]\varphi$$

iff

$$\text{for all set } \{\psi_i : i \in G\} \subseteq \mathcal{L}_{\text{EL}}, M, w \models [\bigwedge_{i \in G} \text{K}_i \psi_i]\varphi$$

- $[\text{K}_i\psi_i]$: 'agent i publicly and truthfully announces ψ_i'.
- Important detail: "the other agents remain silent!"
- The logic can model questions like:
  - 'can group G makes group H knows that φ?'
  - etc.

Group Announcement Logic

Axiomatization

- All principles for PAL
- $[G]\varphi \to [\bigwedge_{i \in G} \mathrm{K}_i \psi_i]\varphi$, where $\psi_i \in \mathcal{L}_{\mathrm{EL}}$
- From $\varphi \to [\chi][\bigwedge_{i \in G} \mathrm{K}_i p_i]\psi$ infer $\varphi \to [\chi][G]\psi$, where p_i does not occur in φ, ψ or χ

Group Announcement Logic

Some Logical Properties

1. $\vdash [\emptyset]\varphi \leftrightarrow \langle\emptyset\rangle\varphi \leftrightarrow \varphi$ ($\emptyset$ is powerless)
2. $\vdash [G]\varphi \rightarrow \varphi$ (T)
3. $\vdash [G \cup H]\varphi \rightarrow [G][H]\varphi$ (4)
4. $\vdash \langle G\rangle[H]\varphi \rightarrow [H]\langle G\rangle\varphi$ (CR)
5. $\vdash \mathrm{K}_i[i]\varphi \leftrightarrow [i]\mathrm{K}_i\varphi$ (agent perfect recall and no-miracles)
6. $\vdash \mathrm{K}_i[G]\varphi \rightarrow [G]\mathrm{K}_i\varphi$ (group perfect recall)
7. $\vdash \langle i\rangle\mathrm{K}_j p \leftrightarrow \langle j\rangle\mathrm{K}_i p$

Group Announcement Logic

Other Properties

- Single-agent GAL is not more expressive than EL.
- Multi-agents GAL is more expressive than EL (and thus, also PAL).
- GAL is not at least as expressive as APAL.
- Model Checking in GAL is PSPACE-complete.

Alternating-time Temporal Announcement Logic

Vocabulary and Notation

- Proposed by [de Lima, CLIMA 2011].
- We assume:
  - a countable set $P = \{p, q, \ldots\}$ of propositional variables;
  - a finite set $N = \{i, j, \ldots\}$ of labels denoting agents;
  - a countable set $A_i = \{\epsilon\} \cup \{a, b, \ldots\}$ of labels denoting the actions available for each agent $i \in N$.
  - where ϵ denotes the *no-operation action* (or skip).
- A *joint action* α is a set of pairs $\{(i, a) : i \in N \text{ and } a \in A_i\}$
- Let $G \subseteq N$. A *partial joint action* α_G is the joint action α with its domain restricted to G, i.e., $\alpha_G = \{(i, a) : i \in G \text{ and } a \in A_i\}$
- A_G denotes the set of all partial joint actions available for group G.
- (Note that $\alpha_N = \alpha$ and also that $\alpha_\emptyset = \emptyset$.)

Alternating-time Temporal Announcement Logic

Language

- Language $\mathcal{L}_{\llbracket\ \rrbracket}$:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{K}_i\varphi \mid \llbracket \alpha_G \rrbracket \varphi$$

  where p ranges over P, i ranges over N, α ranges over A_N and G ranges over 2^N.

- $\langle\!\langle \alpha_G \rangle\!\rangle \varphi \overset{\text{def}}{=} \neg \llbracket \alpha_G \rrbracket \neg\varphi$

Alternating-time Temporal Announcement Logic

Intended Meanings

- $\alpha_G = \{(i_1, a_1), \ldots, (i_{|G|}, a_{|G|})\}$:
  'all the agents in $\{i_1, \ldots, i_{|G|}\}$ execute their corresponding actions in $\{a_1, \ldots a_{|G|}\}$ simultaneously (and we do not consider what the other agents are doing at the same time)'.
- $[\![\alpha_G]\!]\varphi$:
  'after every possible occurrence of α_G, φ is true'.
- $\langle\!\langle\alpha_G\rangle\!\rangle\varphi$:
  'there is an occurrence of α_G after which φ is true'.

Alternating-time Temporal Announcement Logic

Action Descriptions

- We assume an action description $\mathrm{D}(a)$ for each $a \in A_i$ and each $i \in N$.
- $\mathrm{D}(a) = \langle \mathrm{pre}(a), \mathrm{pos}(a) \rangle$, where:
  - $\mathrm{pre}(a) \in \mathcal{L}_{\mathrm{EL}}$ is the executability precondition of a (i.e., a is executable if and only if $\mathrm{pre}(a)$ is true.)
  - $\mathrm{pos}(a) : P \to \mathcal{L}_{\mathrm{EL}}$ is a **partial function** denoting the postconditions of a
    (i.e., p is true after the execution of a if and only if $\mathrm{pos}(a)(p)$ is true before the execution of a.)
- For the action ϵ, we stipulate:

$$\mathrm{pre}(\epsilon) = \top$$
$$\mathrm{pos}(\epsilon) = \emptyset$$

Alternating-time Temporal Announcement Logic

Examples

- The public announcement of φ:

$$\begin{aligned} \mathrm{pre}(a) &= \varphi \\ \mathrm{pos}(a) &= \emptyset \end{aligned}$$

- The public assignment $p := \varphi, q := \psi$

$$\begin{aligned} \mathrm{pre}(a) &= \top \\ \mathrm{pos}(a)(p) &= \varphi \\ \mathrm{pos}(a)(q) &= \psi \end{aligned}$$

Alternating-time Temporal Announcement Logic

Precondition of (Partial) Joint Actions

- Let $\alpha_G = \{(i_1, a_1), \ldots, (i_{|G|}, a_{|G|})\}$.
- The precondition of α_G is:

$$\text{pre}(\alpha_G) = \text{pre}(a_1) \wedge \cdots \wedge \text{pre}(a_{|G|})$$

- In words, joint action α_G is executable if an only if each individual action a_n is executable.

Alternating-time Temporal Announcement Logic

Postconditions of (Partial) Joint Actions

- Let $\alpha_G = \{(i_1, a_1), \ldots, (i_{|G|}, a_{|G|})\}$.
- The postconditions of α_G are:

$$\begin{aligned} \mathrm{pos}(\alpha_G)(p) = \; & (\mathrm{pos}(a_1) \wedge \cdots \wedge \mathrm{pos}(a_{|G|})) \vee \\ & (p \wedge (\mathrm{pos}(a_1) \vee \cdots \vee \mathrm{pos}(a_{|G|}))) \end{aligned}$$

- In words, the truth value of p after the execution of α_G will:
  - be true if every $\mathrm{pos}(a_n)(p)$ is true ;
  - be false if every $\mathrm{pos}(a_n)(p)$ is false;
  - remain the same otherwise.

- The *update* of M by action α_N is the triple $M|\alpha_N = \langle W|\alpha_N, R|\alpha_N, V|\alpha_N \rangle$, where:

$$W|\alpha_N = \{w : M, w \models \mathrm{pre}(\alpha_N)\}$$
$$R|\alpha_N(i) = R(i) \cap (W|\alpha_N \times W|\alpha_N)$$
$$V|\alpha_N(p) = \{w : M, w \models \mathrm{pos}(\alpha_N)(p)\} \cap W|\alpha_N$$

Alternating-time Temporal Announcement Logic

Semantics

$$
\begin{array}{lll}
M, w \models \top & & \\
M, w \models p & \text{iff} & w \in V(p) \\
M, w \models \neg\varphi & \text{iff} & M, w \not\models \varphi \\
M, w \models \varphi \wedge \psi & \text{iff} & M, w \models \varphi \text{ and } M, w \models \psi \\
M, w \models \mathrm{K}_i\varphi & \text{iff} & \text{for all } v \in W, \text{ if } (w, v) \in R(i) \text{ then } M, v \models \varphi
\end{array}
$$

$$
\begin{array}{lll}
M, w \models [\![\alpha_G]\!]\varphi & \text{iff} & \text{for all } \beta_{N\setminus G} \in A_{N\setminus G}, \\
& & \text{if } M, w \models \mathrm{pre}(\alpha_G \cup \beta_{N\setminus G}) \\
& & \text{then } M|(\alpha_G \cup \beta_{N\setminus G}), w \models \varphi
\end{array}
$$

Alternating-time Temporal Announcement Logic

Embedding PAL

- To simulate the public announcement of φ we take some α_G such that:
  - $\text{pre}(\alpha_G) \leftrightarrow \varphi$
  - $\text{pos}(\alpha_G)(p) \leftrightarrow p$, for all $p \in P$
- Then, the announcement of φ is simulated by $\alpha_G \cup \epsilon_{N \setminus G}$, because:

$$
\begin{array}{lll}
M, w \models [\![\alpha_G \cup \epsilon_{N\setminus G}]\!]\psi & \text{iff} & \text{for all } \beta \in A_N \\
 & & \text{if } M, w \models \text{pre}(\alpha_G \cup \epsilon_{N\setminus G} \cup \beta_{N\setminus N}) \\
 & & \text{then } M|(\alpha_N \cup \epsilon_{N\setminus G} \cup \beta_{N\setminus N}), w \models \psi \\
 & \text{iff} & \text{if } M, w \models \text{pre}(\alpha_G) \text{ then } M|\alpha_G, w \models \varphi \\
 & \text{iff} & M, w \models [\alpha_G]\psi
\end{array}
$$

(because $\text{pre}(\epsilon_{N\setminus G}) \leftrightarrow \top$ and $\beta_\emptyset = \emptyset$)

Alternating-time Temporal Announcement Logic

Embedding APAL

- The Arbitrary announcement (and assignment) operator is definable:

$$
\begin{array}{lll}
M, w \models [\![\alpha_\emptyset]\!]\psi & \text{iff} & \text{for all } \beta \in A_N, \\
& & \text{if } M, w \models \mathrm{pre}(\alpha_\emptyset \cup \beta_N) \\
& & \text{then } M|(\alpha_\emptyset \cup \beta_N), w \models \psi \\
& \text{iff} & \text{for all } \beta \in A_N, \\
& & \text{if } M, w \models \mathrm{pre}(\beta_N) \\
& & \text{then } M|\beta_N, w \models \psi \\
& \text{iff} & \text{for all } \beta \in A_N, M, w \models [\beta_N]\psi \\
& \text{iff} & M, w \models \Box\psi
\end{array}
$$

(because $\alpha_\emptyset = \emptyset$)

Alternating-time Temporal Announcement Logic

Embedding GAL

- The group announcement (and assignment) operator is definable:

$$
\begin{array}{lll}
M, w \models [\![\epsilon_{N\setminus G}]\!]\psi & \text{iff} & \text{for all } \beta \in A_N, \\
 & & \text{if } M, w \models \text{pre}(\epsilon_{N\setminus G} \cup \beta_G) \\
 & & \text{then } M|(\epsilon_{N\setminus G} \cup \beta_G), w \models \psi \\
 & \text{iff} & \text{for all } \beta \in A_N, \\
 & & \text{if } M, w \models \text{pre}(\beta_G) \\
 & & \text{then } M|\beta_G, w \models \psi \\
 & \text{iff} & \text{for all } \beta \in A_N, M, w \models [\beta_G]\psi \\
 & \text{iff} & M, w \models [G]\psi
\end{array}
$$

(because $\text{pre}(\epsilon_{N\setminus G}) \leftrightarrow \top$)

Alternating-time Temporal Announcement Logic

Example: Light bulb and light switch

- Irene and Jane live in a strange house: its interior is illuminated by a light bulb, but the switch is located outside the house. Irene is inside the house and Jane is outside it, close to the switch. They want to achieve a state satisfying $\mathrm{K}_i p \wedge \mathrm{K}_j p$.
- Let $N = \{i, j\}$:

$$\mathrm{D}(tog) = \langle \top, \{(p \mapsto \neg p)\} \rangle$$
$$\mathrm{D}(on) = \langle p, \emptyset \rangle$$
$$\mathrm{D}(off) = \langle \neg p, \emptyset \rangle$$

- Let $A_i = \{\epsilon, on, off\}, A_j = \{\epsilon, tog\}$.

Alternating-time Temporal Announcement Logic

Example: Light bulb and light switch (cont.)

- Let some actions be:

$$\alpha_{\{i,j\}} = \{(i, on), (j, \epsilon)\} \qquad \beta_{\{i,j\}} = \{(i, \epsilon), (j, tog)\}$$
$$\alpha'_{\{i,j\}} = \{(i, off), (j, \epsilon)\} \qquad \beta'_{\{i,j\}} = \{(i, \epsilon), (j, \epsilon)\}$$

- We have, for all (M, w):

$$M, w \models p \rightarrow [\![\alpha_{\{i,j\}}]\!][\![\beta'_{\{i,j\}}]\!](\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models p \rightarrow \langle\!\langle \epsilon_j \rangle\!\rangle \langle\!\langle \epsilon_i \rangle\!\rangle (\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models \neg p \rightarrow [\![\alpha'_{\{i,j\}}]\!][\![\beta_{\{i,j\}}]\!](\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models \neg p \rightarrow \langle\!\langle \epsilon_j \rangle\!\rangle \langle\!\langle \epsilon_i \rangle\!\rangle (\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models \langle\!\langle \epsilon_j \rangle\!\rangle \langle\!\langle \epsilon_i \rangle\!\rangle (\mathrm{K}_i p \wedge \mathrm{K}_j p)$$

All principles for multi-agents EL plus:

(AA) $[\![\alpha_N]\!]p \leftrightarrow (\text{pre}(\alpha_N) \rightarrow \text{pos}(\alpha_N)(p))$

(AN) $[\![\alpha_N]\!]\neg\varphi \leftrightarrow (\text{pre}(\alpha_N) \rightarrow \neg[\![\alpha_G]\!]\varphi)$

(AC) $[\![\alpha_N]\!](\varphi \wedge \psi) \leftrightarrow ([\![\alpha_N]\!]\varphi \wedge [\![\alpha_N]\!]\psi)$

(AK) $[\![\alpha_N]\!]\text{K}_i\varphi \leftrightarrow (\text{pre}(\alpha_N) \rightarrow \text{K}_i[\![\alpha_N]\!]\varphi)$

(AS) $([\![\alpha_G]\!]\varphi \wedge [\![\beta_H]\!]\psi) \rightarrow [\![\alpha_G \cup \beta_H]\!](\varphi \wedge \psi) \quad (G \cap H = \emptyset)$

(RA) From $\eta([\![\alpha_G \cup \beta_H]\!]\varphi)$, for all $\beta \in A_N$, infer $\eta([\![\alpha_G]\!]\varphi)$

where η is a necessity form.

Alternating-time Temporal Announcement Logic

Some Interesting Properties

1. If $\vdash \varphi$ then $\vdash [\![\alpha_G]\!]\varphi$ (necessitation)
2. $\vdash [\![\alpha_G]\!]\varphi \rightarrow [\![\alpha_G \cup \beta_H]\!]\varphi$ (outcome monotonicity)
3. $\vdash [\![\alpha_G]\!](\varphi \wedge \psi) \leftrightarrow ([\![\alpha_G]\!]\varphi \wedge [\![\alpha_G]\!]\psi)$ (act. and conjunction)
4. $\vdash \mathrm{K}_i[\![\alpha_G]\!]\varphi \rightarrow [\![\alpha_G]\!]\mathrm{K}_i\varphi$ (perfect recall)

Proof of item 4.

1. for all $\beta \in A_N$, $\vdash ([\![\alpha_G]\!]\varphi \wedge [\![\beta_{N\setminus G}]\!]\top) \rightarrow [\![\alpha_G \cup \beta_{N\setminus G}]\!](\varphi \wedge \top)$ (AS)
2. for all $\beta \in A_N$, $\vdash \mathrm{K}_i[\![\alpha_G]\!]\varphi \rightarrow \mathrm{K}_i[\![\alpha_G \cup \beta_{N\setminus G}]\!]\varphi$ (1 + EL)
3. for all $\beta \in A_N$, $\vdash \mathrm{K}_i[\![\alpha_G]\!]\varphi \rightarrow [\![\alpha_G \cup \beta_{N\setminus G}]\!]\mathrm{K}_i\varphi$ (2 + AK)
4. $\vdash \mathrm{K}_i[\![\alpha_G]\!]\varphi \rightarrow [\![\alpha_G]\!]\mathrm{K}_i\varphi$ (3 + RA)

□

Alternating-time Temporal Announcement Logic

(Un)decidability

- In general, validity checking is not decidable. It follows immediately from the non-decidability of APAL (French and van Ditmarsch, AiML'08).
- However, if A_i is finite, then so is A_N. Then, rule RA can be replaced by the axiom:

$$\text{(RA')} \qquad \bigwedge_{\beta \in A_N} [\![\alpha_G \cup \beta_{N \setminus G}]\!]\varphi \rightarrow [\![\alpha_G]\!]\varphi$$

- Together with outcome monotonicity we have:

$$\vdash [\![\alpha_G]\!]\varphi \leftrightarrow \bigwedge_{\beta \in A_N} [\![\alpha_G \cup \beta_{N \setminus G}]\!]\varphi$$

and, therefore, it can be reduced to epistemic logic.

Alternating-time Temporal Announcement Logic

Coalition Operator

- The language $\mathcal{L}_X$ is defined by the BNF:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{K}_i\varphi \mid [\![\alpha_G]\!]\varphi \mid \langle\!\langle G \rangle\!\rangle\varphi$$

- $[\![G]\!]\varphi \overset{\text{def}}{=} \neg\langle\!\langle G \rangle\!\rangle\neg\varphi$
- Intended meanings:
  - $\langle\!\langle G \rangle\!\rangle\varphi$:
    'group G is able to enforce that φ is true in the next step'
  - $[\![G]\!]\varphi$:
    'group G is not able to avoid that φ is true in the next step'
- Interpretation:

  $M, w \models \langle\!\langle G \rangle\!\rangle\varphi$

  iff

  there is $\alpha \in A_N$ such that $M, w \models \neg[\![\alpha_G]\!]\bot$ and $M, w \models [\![\alpha_G]\!]\varphi$

Alternating-time Temporal Announcement Logic

Axiomatization and (Un)decidability

- All the principles seen before plus:

(AG) $(\langle\langle \alpha_G \rangle\rangle \top \wedge [\![\alpha_G]\!]\varphi) \rightarrow \langle\langle G \rangle\rangle \varphi$

(RG) From $\eta([\![\alpha_G]\!]\bot \wedge \langle\langle \alpha_G \rangle\rangle \varphi)$, for all $\alpha \in A_N$, infer $\eta([\![G]\!]\varphi)$

- In general, validity checking is not decidable.
- But, if A_i is finite for all $i \in N$, then:

(RG') $$\bigwedge_{\alpha \in A_N} ([\![\alpha_G]\!]\bot \wedge \langle\langle \alpha_G \rangle\rangle \varphi) \rightarrow [\![G]\!]\varphi$$

and, in this case, it can be reduced to epistemic logic.

Alternating-time Temporal Announcement Logic

Some Interesting Properties

1. $\vdash \langle\!\langle G \rangle\!\rangle \top$ (group activity)
2. $\vdash \neg \langle\!\langle G \rangle\!\rangle \bot$ (group non-blocking)
3. $\vdash \neg \langle\!\langle \emptyset \rangle\!\rangle \neg \varphi \to \langle\!\langle N \rangle\!\rangle \varphi$ (joint determinism)
4. $\vdash (\langle\!\langle G \rangle\!\rangle \varphi \wedge \langle\!\langle H \rangle\!\rangle \psi) \to \langle\!\langle G \cup H \rangle\!\rangle (\varphi \wedge \psi)$ (group superadd.)
5. If $\vdash \varphi \to \psi$ then $\vdash \langle\!\langle G \rangle\!\rangle \varphi \to \langle\!\langle G \rangle\!\rangle \psi$ (monotonicity)

(Note that these are the axioms of Coalition Logic.)

Alternating-time Temporal Announcement Logic

Example: Light bulb and light switch (cont.)

- We have, for all (M, w):

$$M, w \models p \rightarrow [\![\alpha_{\{i\}}]\!]\langle\!\langle\{j\}\rangle\!\rangle(\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models p \rightarrow \langle\!\langle\{i\}\rangle\!\rangle\langle\!\langle\{j\}\rangle\!\rangle(\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models \neg p \rightarrow [\![\alpha'_{\{i\}}]\!]\langle\!\langle\{j\}\rangle\!\rangle(\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models \neg p \rightarrow \langle\!\langle\{i\}\rangle\!\rangle\langle\!\langle\{j\}\rangle\!\rangle(\mathrm{K}_i p \wedge \mathrm{K}_j p)$$
$$M, w \models \langle\!\langle\{i\}\rangle\!\rangle\langle\!\langle\{j\}\rangle\!\rangle(\mathrm{K}_i p \wedge \mathrm{K}_j p)$$

Alternating-time Temporal Announcement Logic

Temporal Operators

- Language $\mathcal{L}_{\text{ATAL}}$:

  $\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathrm{K}_i\varphi \mid [\![\alpha_G]\!]\varphi \mid \langle\!\langle G \rangle\!\rangle\varphi \mid \langle\!\langle G \rangle\!\rangle^*\varphi \mid \langle\!\langle G, \varphi \rangle\!\rangle\varphi$

- Intended meanings:
  - $\langle\!\langle G \rangle\!\rangle^*\varphi$:
    'group G is able to enforce that φ is true from now on'.
  - $\langle\!\langle G, \psi \rangle\!\rangle\varphi$:
    'group G is able to enforce that eventually φ will be true, while meanwhile enforcing that ψ is true'.
- Interpretation:

$$
\begin{array}{lll}
M, w \models \langle\!\langle G \rangle\!\rangle^*\varphi & \text{iff} & \text{for all } n \geq 0, M, w \models \langle\!\langle G \rangle\!\rangle^n\varphi \\
M, w \models \langle\!\langle G, \psi \rangle\!\rangle\varphi & \text{iff} & \text{there is } n \text{ such that } n \geq 0 \text{ and } M, w \models \langle\!\langle G \rangle\!\rangle^n\varphi \\
 & & \text{and for all } m \text{ if } 0 < m \leq n \text{ then } M, w \models \langle\!\langle G \rangle\!\rangle^m\psi
\end{array}
$$

where $\langle\!\langle G \rangle\!\rangle^0\varphi \stackrel{\text{def}}{=} \varphi$ and $\langle\!\langle G \rangle\!\rangle^{n+1}\varphi \stackrel{\text{def}}{=} \langle\!\langle G \rangle\!\rangle\langle\!\langle G \rangle\!\rangle^n\varphi$.

Alternating-time Temporal Announcement Logic

Axiomatization and (Un)decidability

- Completeness has been achieved only for the case with a **finite number of actions**.
- Axiomatization: all the principles seen before plus:

(FPA) $\langle\langle G \rangle\rangle^* \varphi \to (\varphi \wedge \langle\langle G \rangle\rangle \langle\langle G \rangle\rangle^* \varphi)$

(FPU) $\langle\langle G, \psi \rangle\rangle \varphi \to (\varphi \vee (\psi \wedge \langle\langle G \rangle\rangle \langle\langle G, \psi \rangle\rangle \varphi))$

(RIA) From $\chi \to (\varphi \wedge \langle\langle G \rangle\rangle \chi)$ infer $\chi \to \langle\langle G \rangle\rangle^* \varphi$

(RIU) From $(\varphi \vee (\psi \wedge \langle\langle G \rangle\rangle \chi)) \to \chi$ infer $\langle\langle G, \psi \rangle\rangle \varphi \to \chi$

- For a finite number of actions, validity checking is also decidable.

Alternating-time Temporal Announcement Logic

Example: Light bulb and light switch (cont.)

- We have, for all (M, w):

$$M, w \models \langle\!\langle \{i\} \rangle\!\rangle \langle\!\langle \{j\} \rangle\!\rangle (\mathrm{K}_i p \wedge \mathrm{K}_j p)$$

$$M, w \models \langle\!\langle \{i, j\}, \top \rangle\!\rangle (\mathrm{K}_i p \wedge \mathrm{K}_i p)$$

$$M, w \models \langle\!\langle \{i, j\}, \top \rangle\!\rangle \langle\!\langle \{i, j\} \rangle\!\rangle^* (\mathrm{K}_i p \wedge \mathrm{K}_j p)$$

Alternating-time Temporal Announcement Logic

Some Perspectives

- Model checking
- Computational complexity
- Complex actions (strategies, multi-agents planning)
- Preferences (Nash equilibrium, dominance, etc.)

$$\alpha_i \mathrm{BR} \beta_j \stackrel{\mathrm{def}}{=} \langle\!\langle \alpha_i \cup \beta_j \rangle\!\rangle \neg\varphi_i \rightarrow [\![\beta_j]\!] \neg\varphi_i$$

$$\mathrm{NE}(\alpha_i, \beta_j) \stackrel{\mathrm{def}}{=} (\alpha_i \mathrm{BR} \alpha_j) \wedge (\beta_j \mathrm{BR} \alpha_i)$$

- Group and common knowledge
- Private actions, suspicions (communication protocols): Alternating-time Temporal Dynamic Epistemic Logic

Thank you!