Global and local graph modifiers

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Background: Dynamic Epistemic Logics (*DEL*)

The *DEL* interpretation of modalities

- graph = pointed model $M = \langle W, w, R, V \rangle$
- traditionally: modality = relation between nodes

$$\langle W, w, R, V \rangle \models [\alpha]\varphi \text{ iff } \forall w' \in R(\alpha)(w),\ \langle W, w', R, V \rangle \models \varphi$$

- *DEL*: modality = (partial) function on the set of pointed models

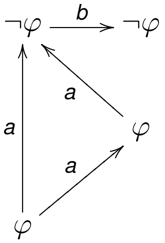
$$M \models [\alpha]\varphi \text{ iff } \mathcal{R}_\alpha(M) \text{ defined implies } \mathcal{R}_\alpha(M) \models \varphi$$

- here: generalize existing *DEL*s: modalities as graph modifiers
  - delete/add nodes, delete/add edges, ...
  - global vs. local modifiers: 'modify everywhere' vs. 'modify at the actual world'
  - existence of reduction axioms

Example: public announcements, version of [Kooi 07]

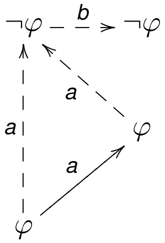
- context: logic of belief
- $\varphi!_K$ = "it is publicly announced to all agents that φ"
  - delete all edges leading to $\neg\varphi$-worlds
- for $M = \langle W, w, R, V\rangle$:
  - $\mathcal{R}_{\varphi!_K}(M) = \langle W, w, R', V\rangle$, where $R'(a) = R(a) \cap (W \times \|\varphi\|_M)$

(M):



$\Longrightarrow$

$\mathcal{R}_{\varphi!}(M)$:



Example: public announcements [Plaza 89]

- context: logic of knowledge
- $\varphi!$ = "it is publicly announced to all agents that φ"
  - delete all $\neg\varphi$-worlds (and the entering and leaving edges)
- for $M = \langle W, w, R, V \rangle$:
  - restriction $\mathcal{R}_{\varphi!}(M) = \langle W', w, R', V' \rangle$ where
    - ★ $W' = \|\varphi\|_M$
    - ★ $R'(a) = R(a) \cap (\|\varphi\|_M \times \|\varphi\|_M)$
    - ★ $V'(p) = V(p) \cap \|\varphi\|_M$

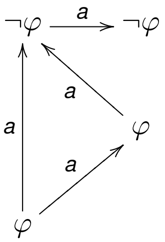
Example: public assignments [Ditmarsch et al. 05]

- extension of *PAL*
- $p := \varphi$
  - "publicly assign value of φ to p"
  - change all valuations such that p takes the value of φ
- for $M = \langle W, w, R, V \rangle$:
  - $\mathcal{R}_{p:=\varphi}(M) = \langle W, w, R, V' \rangle$ where
    - ★ $V'(p) = \|\varphi\|_M$
    - ★ $V'(q) = V(q)$ for $q \neq p$

Example: preference upgrade [Benthem and Liu 07]

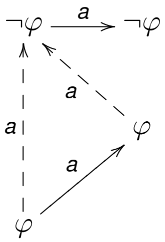
- context: logic of an agent's preferences
- $v \in R(a)(w)$ = "agent a prefers v to w"
- $\#\varphi$ = "make agent prefer that φ"
  - remove all edges from φ-states to $\neg\varphi$-states

(M):



$\Longrightarrow$

$\mathcal{R}_{\varphi!}(M)$:



Example: rights management [Pucella and Weissman 04]

- context: logic of obligation and permissions
- $a+(\varphi, \psi)$ = "in context φ, grant permission that ψ"
  - add all edges from φ-states to ψ-states to $R(a)$
- $a-(\varphi, \psi)$
  - "in context φ, revoke permission that ψ"
  - remove all edges from φ-states to ψ-states from $R(a)$

A generalization: graph modifiers

how can we modify a labelled graph?

- add/delete nodes
- add/delete edges
- add/delete node labels

principle:

- refer to nodes by formulas
- refer to edges by couples of formulas

the scope of the modification:

- global: delete all edges leading from φ-worlds to ψ-worlds
- local: delete all edges leading *from the actual world* to ψ-worlds

Overview

1. Background
2. The logic of global graph modifiers
3. The logic of local graph modifiers

The logic of global graph modifiers

Language

$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid [\alpha]\varphi$

$\alpha ::=$	a	(ranges over *REL*)
	$\alpha ; \alpha$	
	$\varphi ?$	
	$\mathtt{U}$	(universal modal operator)
	$\mathtt{nw}$	"add new node"
	$\overrightarrow{\mathtt{nw}}$	"add new node and go there"
	$p - \varphi$	"delete *p* where φ holds"
	$p + \varphi$	"…"
	$a - (\varphi, \psi)$	"delete edges from φ-worlds to ψ-worlds to *R*(*a*)"
	$a + (\varphi, \psi)$	"…"

N.B.: node deletion to be defined later

Semantics

- pointed models $M = \langle W, w, R, V \rangle$, where $R(a) \subseteq W \times W$
- truth condition:

$$M \models [\alpha]\varphi \text{ iff } M' \models \varphi, \text{ for every } M' \text{ such that } M \xrightarrow{\alpha} M'$$

- def. relation between models $\langle W, w, R, V \rangle \xrightarrow{\alpha} \langle W', w', R', V' \rangle$
  - for the standard connectors:

$$\begin{array}{ll} \alpha = a: & W' = W, \langle \mathbf{w}, \mathbf{w}' \rangle \in \mathbf{R}(\mathbf{a}), R' = R, V' = V; \\ \alpha = \alpha_1; \alpha_2: & \exists \mathbf{M}'' : \langle \mathbf{W}, \mathbf{w}, \mathbf{R}, \mathbf{V} \rangle \xrightarrow{\alpha_1} \mathbf{M}'' \text{ and } \mathbf{M}'' \xrightarrow{\alpha_2} \langle \mathbf{W}', \mathbf{w}', \mathbf{R}', \mathbf{V}' \rangle; \\ \alpha = \varphi?: & W' = W, w' = w, R' = R, V' = V, \text{ and } \mathbf{M} \models \varphi; \\ \alpha = \mathtt{U}: & W' = W, \mathbf{w}' \in \mathbf{W}, R' = R, V' = V; \end{array}$$

Semantics, ctd.

- def. of $\langle W, w, R, V\rangle \xrightarrow{\alpha} \langle W', w', R', V'\rangle$
  - for the new connectors:

$$
\begin{array}{ll}
\mathtt{nw}: & W' = W \cup \{w_{new}\}, w_{new} \notin W, w' = w, R' = R, V' = V; \\
\overrightarrow{\mathtt{nw}}: & W' = W \cup \{w_{new}\}, w_{new} \notin W, w' = w_{new}, R' = R, V' = V \\
p{-}\varphi: & W' = W, w' = w, R' = R, V'(q) = V(q) \text{ for } q \neq p, \\
 & V'(p) = V(p) \setminus \|\varphi\|_M; \\
p{+}\varphi: & \dots \\
a{-}(\varphi,\psi): & W' = W, w' = w, R'(b) = R(b) \text{ for } b \neq a, \\
 & R'(a) = R(a) \setminus (\|\varphi\|_M \times \|\psi\|_M), V' = V; \\
a{+}(\varphi,\psi): & \dots
\end{array}
$$

- validity, satisfiability defined as usual
- graph modifier logic *GML*

GML contains Kooi's *PAL*

if $REL = \{a_1, \ldots, a_n\}$:

$$[\varphi!_K]\psi \;=\; [a_1 - (\top, \neg\varphi); \ldots ; a_n - (\top, \neg\varphi)]\psi$$

GML contains Plaza's *PAL*

S = set of atoms encoding the worlds deleted by announcements:

$$\begin{array}{lcl} \tau_S(p) & = & p \\ \tau_S(\neg\varphi) & = & \neg\tau_S(\varphi) \\ \tau_S(\varphi \vee \psi) & = & \tau_S(\varphi) \vee \tau_S(\psi) \\ \tau_S([a]\varphi) & = & [a]((\bigwedge S) \to \tau_S(\varphi)) \\ \tau_S([\psi!]\varphi) & = & \tau_S(\psi) \to [p{-}\top][p{+}\psi](p \to \tau_{S\cup\{p\}}(\varphi)), \text{ for } p \text{ new} \end{array}$$

Theorem

$M \models \varphi$ iff $M \models \tau_\emptyset(\varphi)$.

Reduction axioms for *GML* (1)

- modifiers against atoms:

$$
\begin{array}{rcll}
[\mathtt{nw}]p & \leftrightarrow & p & \\
[\overrightarrow{\mathtt{nw}}]p & \leftrightarrow & \bot & \\
[p{-}\varphi]q & \leftrightarrow & q & \text{if } q \neq p \\
 & \leftrightarrow & p \wedge \neg\varphi & \text{else} \\
[p{+}\varphi]q & \leftrightarrow & q & \text{if } q \neq p \\
 & \leftrightarrow & p \vee \varphi & \text{else} \\
[a{-}(\varphi,\psi)]p & \leftrightarrow & p & \\
[a{+}(\varphi,\psi)]p & \leftrightarrow & p &
\end{array}
$$

Reduction axioms for *GML* (2)

- modifiers against booleans:

$$
\begin{array}{rcl}
[\alpha](\chi \wedge \chi') & \leftrightarrow & [\alpha]\chi \wedge [\alpha]\chi' \\
[\alpha](\chi \vee \chi') & \leftrightarrow & [\alpha]\chi \vee [\alpha]\chi' \\
[\alpha]\neg\chi & \leftrightarrow & \neg[\alpha]\chi
\end{array}
$$

Reduction axioms for *GML* (3)

- node creators against standard modalities:

$$
\begin{array}{lcl}
[\mathtt{nw}][a]\chi & \leftrightarrow & [a][\mathtt{nw}]\chi \\
[\mathtt{nw}][\mathtt{U}]\chi & \leftrightarrow & [\overrightarrow{\mathtt{nw}}]\chi \wedge [\mathtt{U}][\mathtt{nw}]\chi \\
[\overrightarrow{\mathtt{nw}}][a]\chi & \leftrightarrow & \top \\
[\overrightarrow{\mathtt{nw}}][\mathtt{U}]\chi & \leftrightarrow & [\overrightarrow{\mathtt{nw}}]\chi \wedge [\mathtt{U}][\mathtt{nw}]\chi
\end{array}
$$

Reduction axioms for *GML* (4)

- node modifiers against standard modalitites:

$$
\begin{array}{lcl}
[p-\varphi][a]\chi & \leftrightarrow & [a][p-\varphi]\chi \\
[p-\varphi][\mathrm{U}]\chi & \leftrightarrow & [\mathrm{U}][p-\varphi]\chi \\
[p+\varphi][a]\chi & \leftrightarrow & [a][p+\varphi]\chi \\
[p+\varphi][\mathrm{U}]\chi & \leftrightarrow & [\mathrm{U}][p+\varphi]\chi
\end{array}
$$

Reduction axioms for *GML* (5)

- edge modifiers against standard modalitites:

$$
\begin{array}{lcll}
[b-(\varphi,\psi)][a]\chi & \leftrightarrow & [a][b-(\varphi,\psi)]\chi & \text{if } b \neq a \\
 & \leftrightarrow & (\neg\varphi \land [a][a-(\varphi,\psi)]\chi) \lor (\varphi \land [a](\neg\psi \to [a-(\varphi,\psi)]\chi) & \text{else} \\
[b-(\varphi,\psi)][\mathrm{U}]\chi & \leftrightarrow & [\mathrm{U}][b-(\varphi,\psi)]\chi & \\
 & & & \\
[b+(\varphi,\psi)][a]\chi & \leftrightarrow & [a][b+(\varphi,\psi)]\chi & \text{if } b \neq a \\
 & \leftrightarrow & [a][a+(\varphi,\psi)]\chi \land (\varphi \to [\mathrm{U}](\psi \to [a+(\varphi,\psi)]\chi) & \text{else} \\
[b+(\varphi,\psi)][\mathrm{U}]\chi & \leftrightarrow & [\mathrm{U}][b+(\varphi,\psi)]\chi &
\end{array}
$$

Reduction axioms for *GML* (6)

Theorem

For every formula φ there is a formula φ' without graph modifiers such that $\varphi \leftrightarrow \varphi'$ is valid.

The logic of local graph modifiers

Language

- language of *GML*, plus local state label modifiers:
  - $p{+}\varphi_{loc}$ = "make p true in the actual world if φ holds there"
  - $p{-}\varphi_{loc}$ = ...
- no local edge label modifiers

Semantics

transition relations $\langle W, w, R, V\rangle \xrightarrow{\alpha} \langle W', w', R', V'\rangle$ for local modifiers:

$\alpha = p{+}\varphi_{loc}$: $W' = W, w' = w, R' = R$, *and*
$w \in V'(p)$ iff $w \in V(p)$ or $w \in \|\varphi\|_M$,
$w \in V'(q)$ iff $w \in V(q)$, for $q \neq p$,
$v \in V'(q)$ iff $v \in V(q)$, for all other $v \neq w$;

$\alpha = p{-}\varphi_{loc}$: $\ldots$

local graph modifier logic *locGML*

Expressivity

- irreflexivity of $R(a)$:

$$M \models [\Cup][p-\top][p+\top_{loc}][a]\neg p$$

- determinism: ...
- converse: ...
- complement: ...
- graded modalities: ...

There are no reduction axioms for *locGML*:

Theorem

The formula $[\Cup][p-\top][p+\top_{loc}][a]\neg p$ is not definable in GML.

Translation from $\mathcal{H}(\mathtt{U}, @, \downarrow)$ into *locGML*

φ_0 formula of hybrid logic with binder $\mathcal{H}(\mathtt{U}, @, \downarrow)$
for every nominal i, a new p_i not occurring in φ_0
for every variable x, a new p_x not occurring in φ_0

$$
\begin{array}{lcl}
\tau(p) & = & p \\
\tau(i) & = & p_i \\
\tau(x) & = & p_x \\
\tau(\neg\varphi) & = & \neg\tau(\varphi) \\
\tau(\varphi \vee \psi) & = & \tau(\varphi) \vee \tau(\psi) \\
\tau([a]\varphi) & = & [a]\tau(\varphi) \\
\tau([\mathtt{U}]\varphi) & = & [\mathtt{U}]\tau(\varphi) \\
\tau(@_i\varphi) & = & \langle\mathtt{U}\rangle(p_i \wedge \tau(\varphi)) \\
\tau(@_x\varphi) & = & \langle\mathtt{U}\rangle(p_x \wedge \tau(\varphi)) \\
\tau(\downarrow x.\varphi) & = & [p_x - \top][p_x + \top_{loc}]\tau(\varphi)
\end{array}
$$

locGML is undecidable

Lemma

φ_0 $\mathcal{L}_{\mathcal{H}(U,@,\downarrow)}$-formula;
$M = \langle W, w, R, V\rangle$ be a $\mathcal{H}(U, @, \downarrow)$-model, g assignment of variables;
$M' = \langle W', w', R', V'\rangle$ locGML-model corresponding with M and g:

- $W' = W$, $w = w'$, $a = a'$,
- $V'(p) = V(p)$ *for every p occurring in* φ_0,
- $V'(p_i) = V(i)$ *for* $i \in NOM$, *and*
- $V'(p_x) = g(x)$ *for* $x \in SVAR$.

Then

$$M, g \models \varphi_0 \text{ iff } M' \models \tau(\varphi_0)$$

Corollary

The logic locGML is undecidable.

Related work on local modifiers: [Renardel de Lavalette 04]

- local state label assignments $p{:=}\varphi_{loc}$
- local edge label assignments $a{:=}\alpha_{loc}$
- Fagin and Vardi's semantics, $\neq$ standard Kripke models
  - reduction axioms for the local modalities, in particular

$$
\begin{array}{lll}
[p{:=}\varphi_{loc}][a]\psi & \leftrightarrow & [a]\psi \\
[b{:=}\alpha_{loc}][a]\psi & \leftrightarrow & [\alpha]\psi \text{ if } b = a \\
 & \leftrightarrow & [a]\psi \text{ else}
\end{array}
$$

  - not valid in our *locGML*

Related work on local modifiers: [Van Benthem 00, Loding and Rohde 03]

sabotage modal operators $a{-}\exists$: locally delete an arbitrary a-edge

- $\langle W, w, R, V\rangle \xrightarrow{a-\exists} \langle W', w', R', V'\rangle$ iff
  - $W' = W$,
  - there is $w' \in W$ such that $\langle w, w'\rangle \in R(a)$,
  - $R' = R \setminus \{\langle w, w'\rangle\}$,
  - $V' = V$.

$[a{-}\exists]\varphi$ =

$$[here{-}\top][here{+}\top_{loc}]\langle a\rangle[there{-}\top][there{+}\top_{loc}][a{-}(here, there)]\varphi$$

here and *there* = fresh labels

Conclusion

- global graph modifiers = nice generalization of dynamic epistemic logics
- global + local graph modifiers = very expressive logic
  - decidable fragments?